SEEING GALAXY CLUSTERS THROUGH COSMIC MICROWAVE BACKGROUND

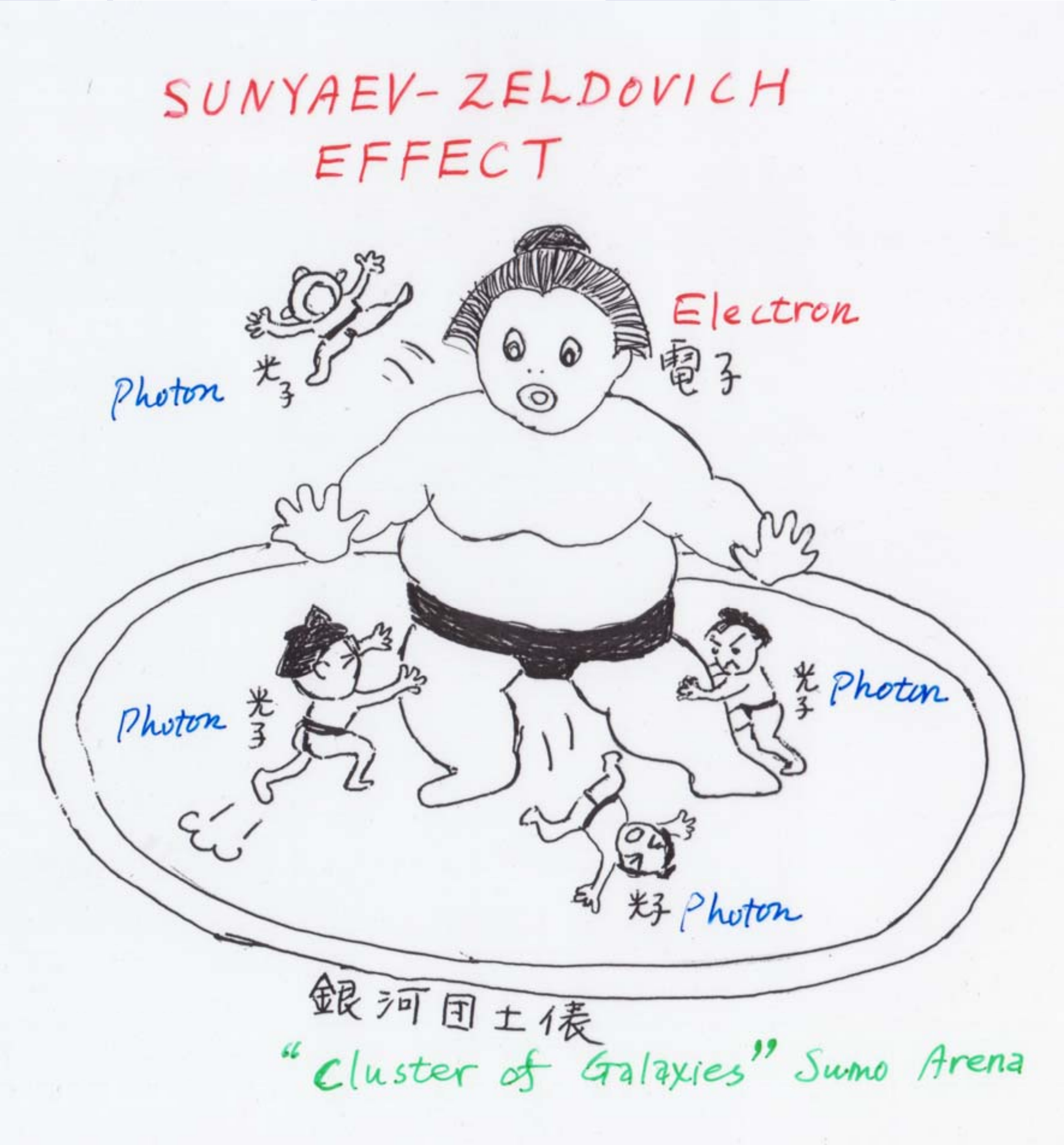
NAOKI ITOH

SOPHIA UNIVERSITY

COLLABORATORS

- SATOSHI NOZAWA
- YASUHARU KOHYAMA
- YOUHEI KAWANA
- YASUHIKO SUDA
- YOICHI OHHATA

SZ EFFECT DRAWN BY YURIKO ITOH



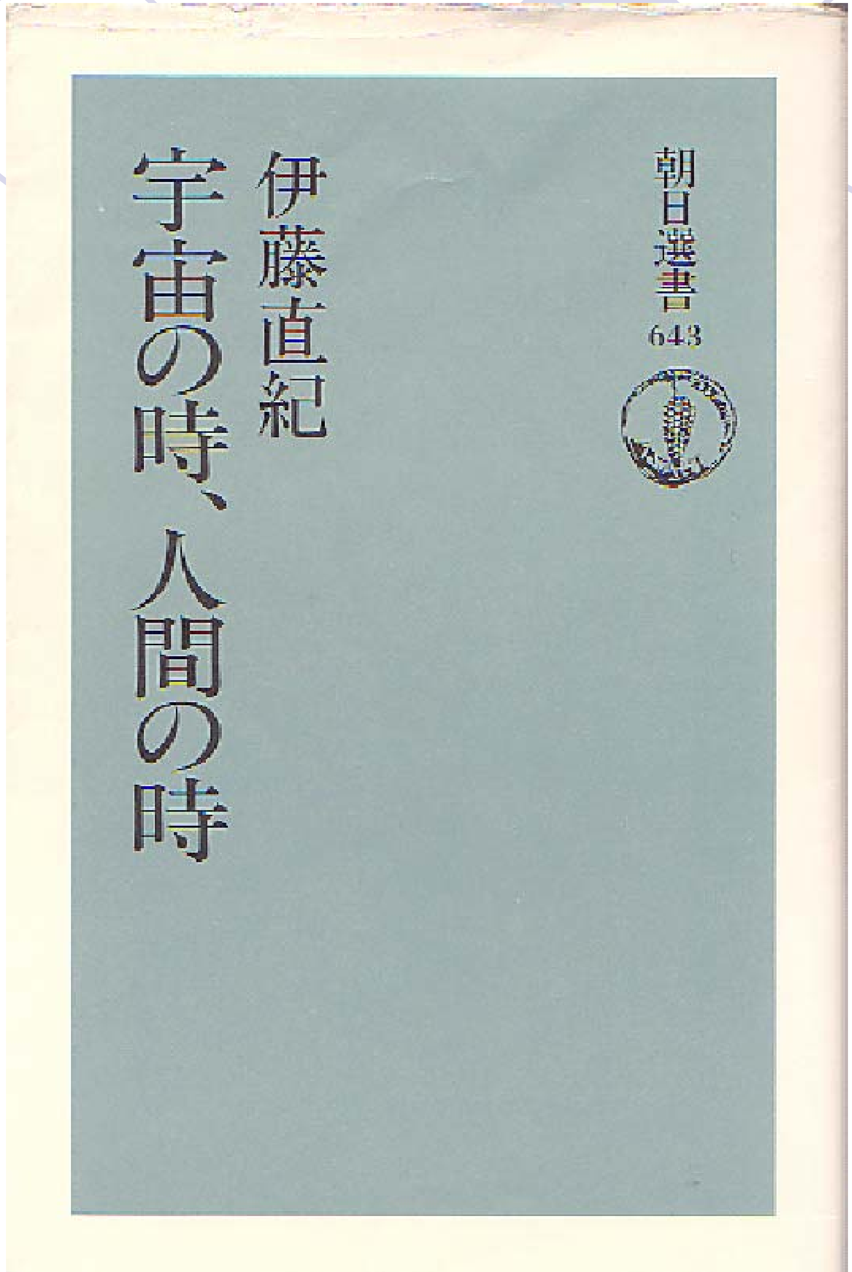

八ヶ岳カンティフェア
KIYOSATO ANNUAL COUNTY FAIR
第14回
八ヶ岳カンティフェア
'87.8.14.15
BOSE
八ヶ岳カンティフェア
八ヶ岳カンティフェア

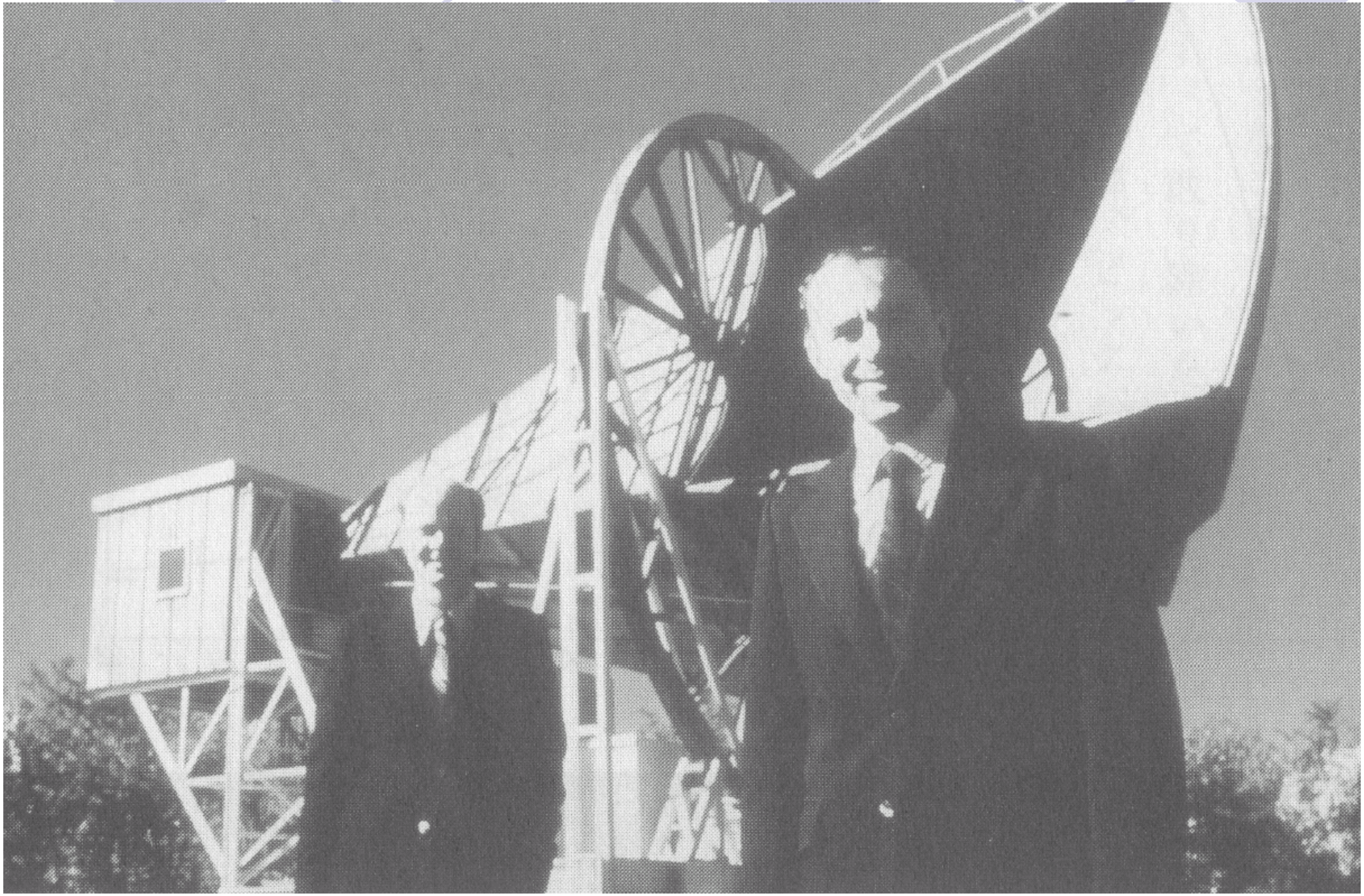

KIYOSATO
ANNUAL
COUNTY
FAIR
BOSE

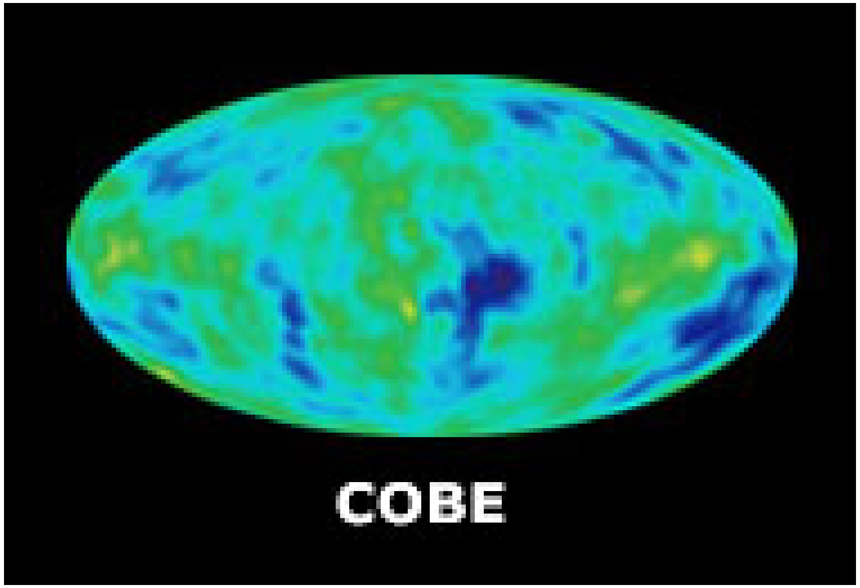

PAUL RUSCH FESTIVAL YATSUGATAKE COUNTY FAIR '92
PAUL RUSCH FESTIVAL YATSUGATAKE
HEART OF HARVEST
八ヶ岳の麓、大草原の収穫感謝祭

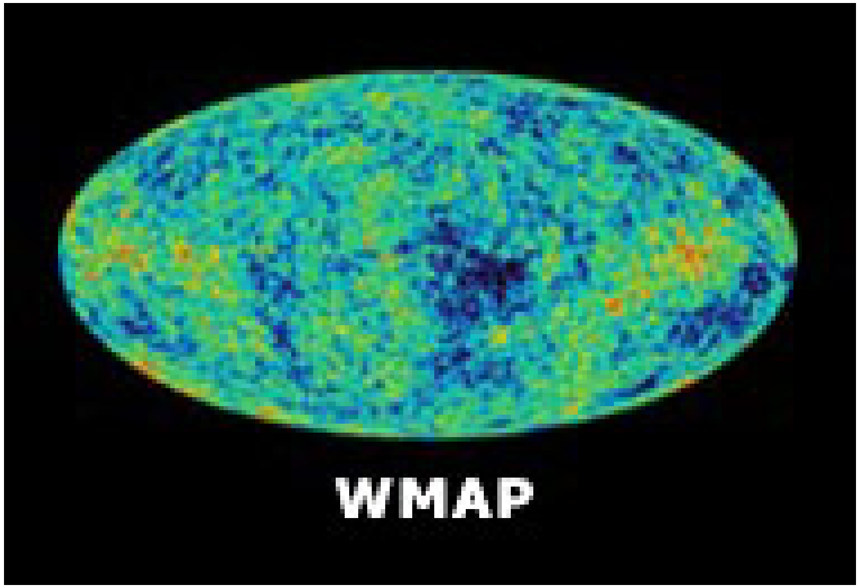

消火器

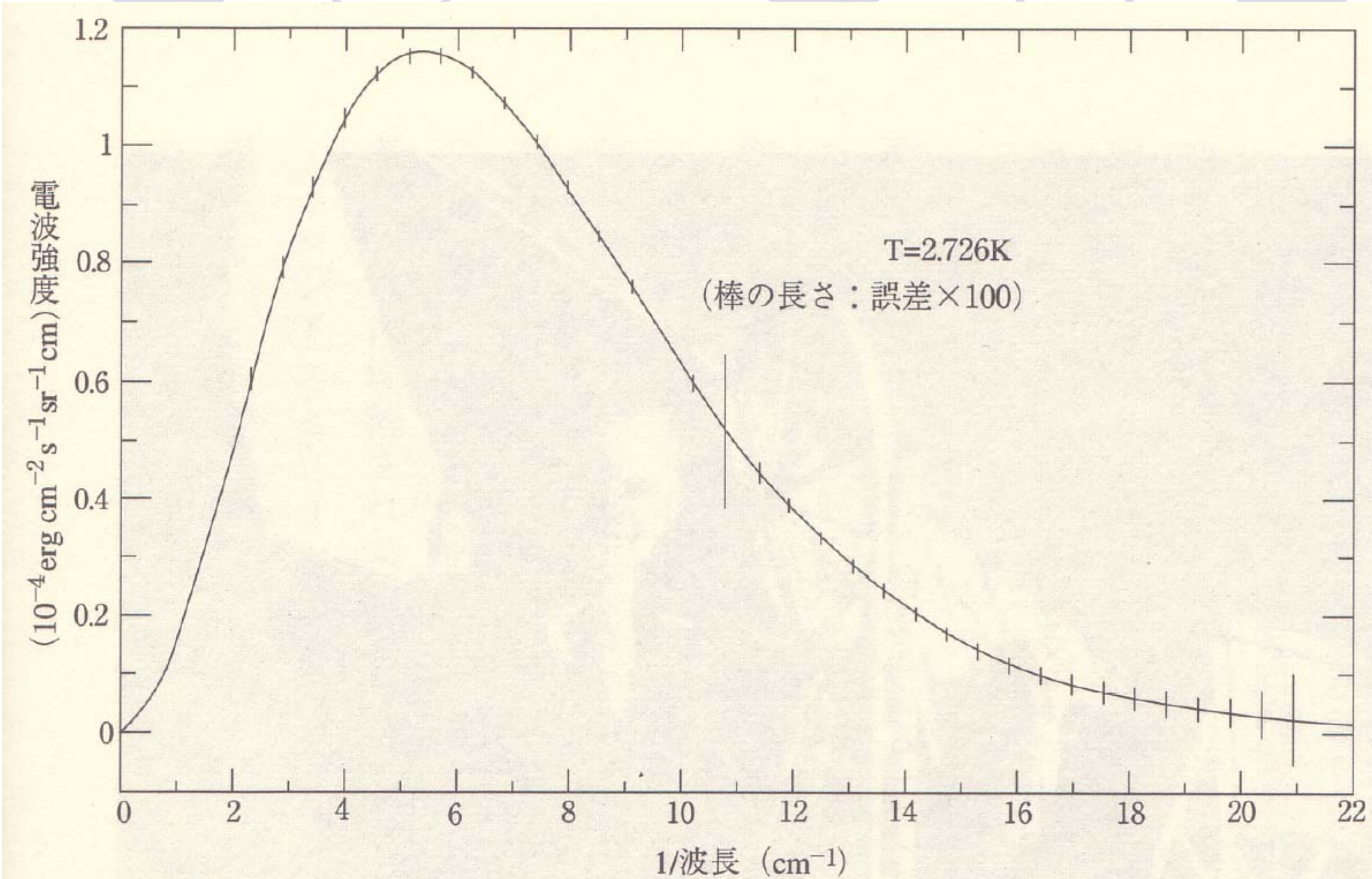

SEISE -RYO
ANFORD


朝日選書
643
伊藤直紀
宇宙の時、人間の時




COBE

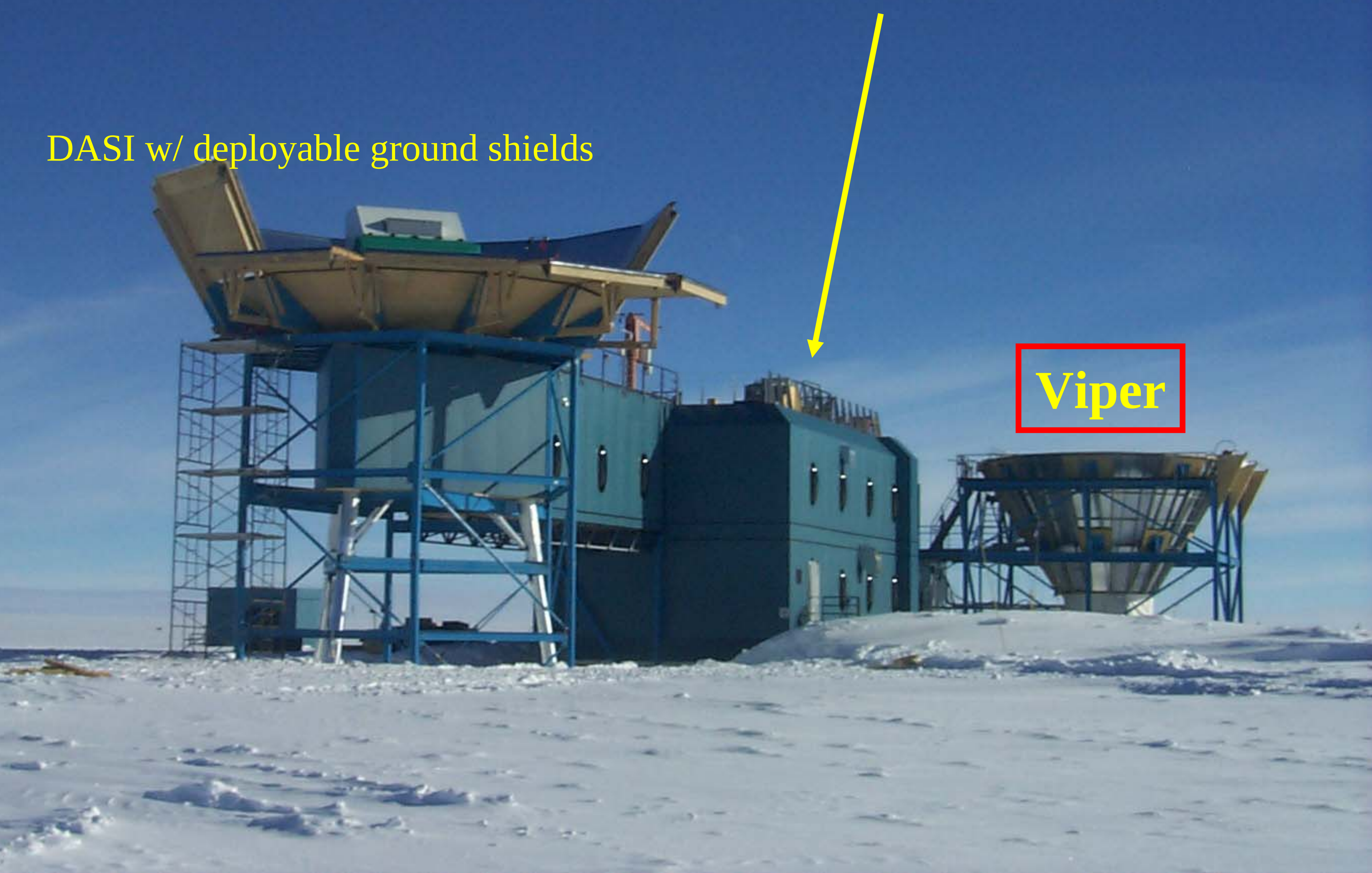

WMAP


T=2.726K
（棒の長さ：誤差×100）
電波強度（10^{-4} erg cm^{-2} s^{-1} sr^{-1} cm）
1/波長（cm^{-1}）
0
0.2
0.4
0.6
0.8
1
1.2
0
2
4
6
8
10
12
14
16
18
20
22

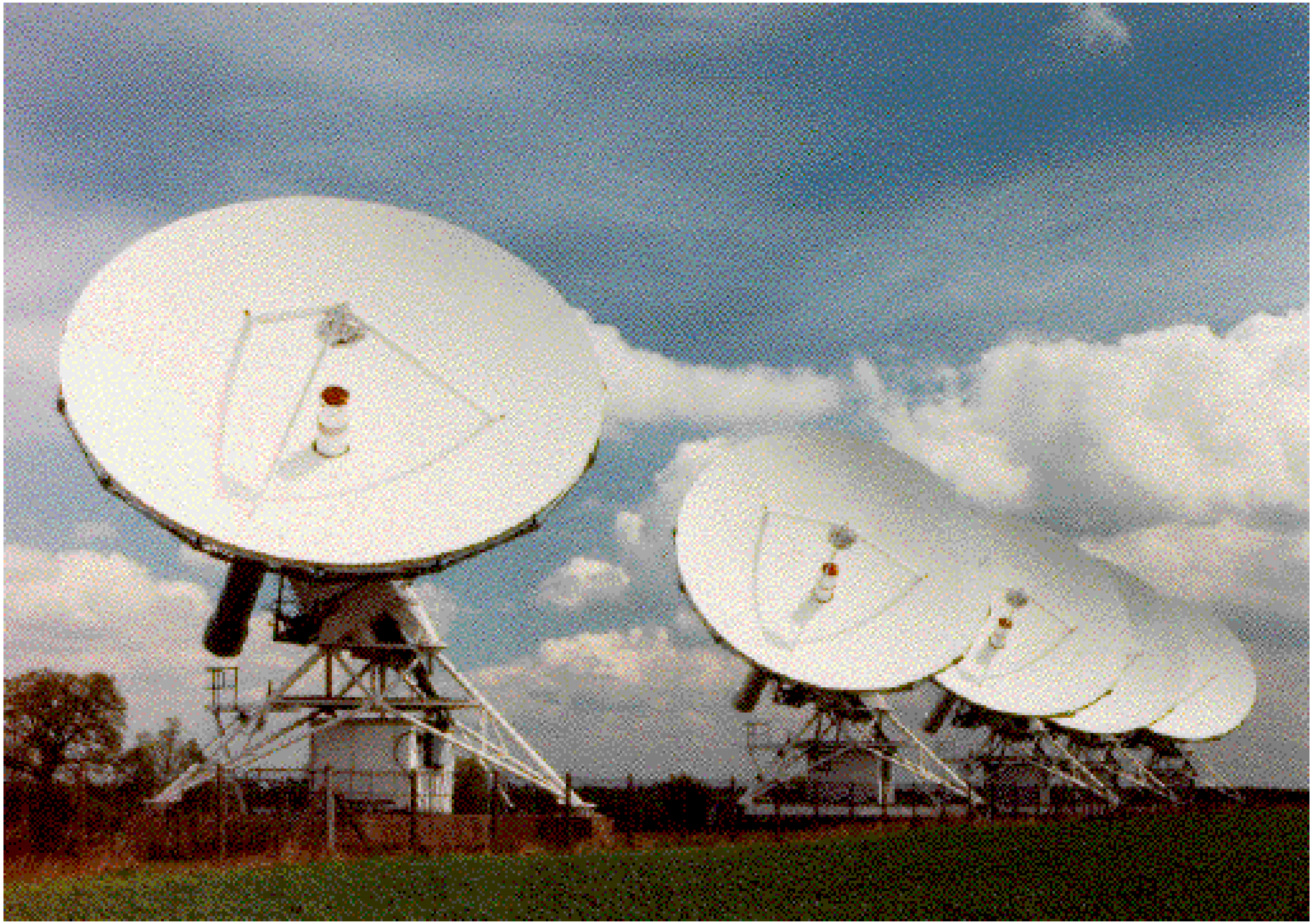
Nobeyama Radio Observatory

Martin A. Pomerantz Observatory at the South Pole

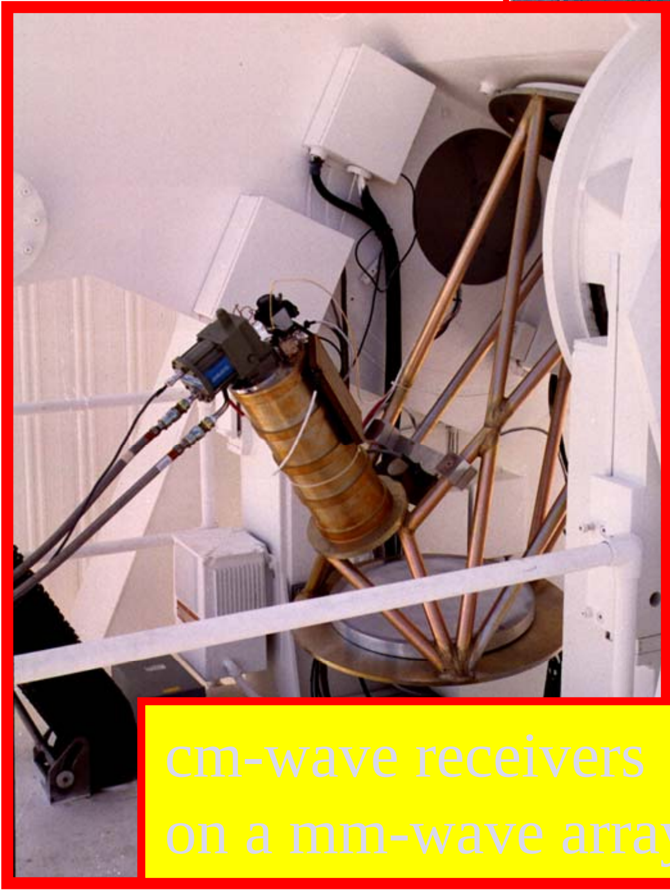
Fully equipped modern lab at South Pole station



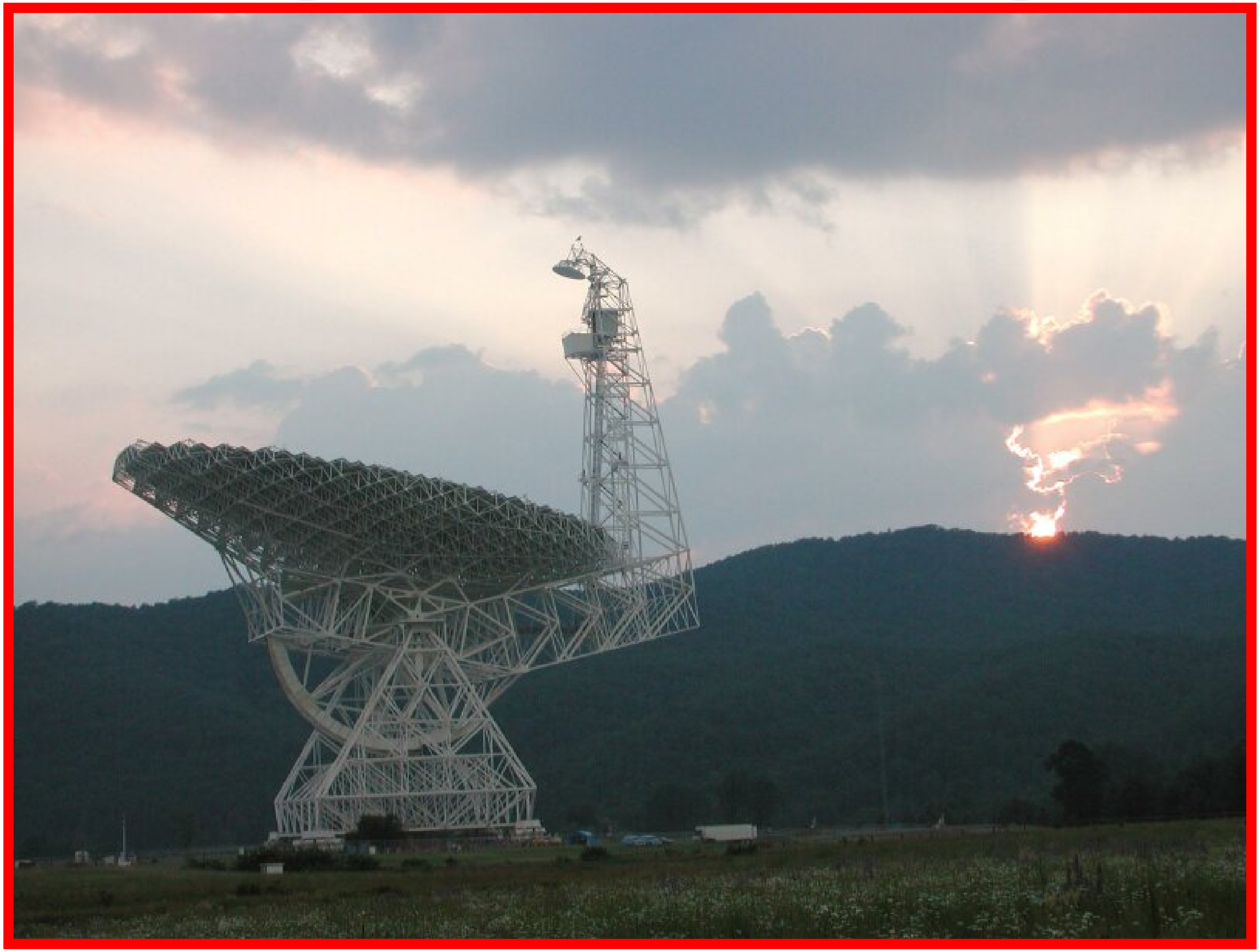
Ryle Telescope MRAO



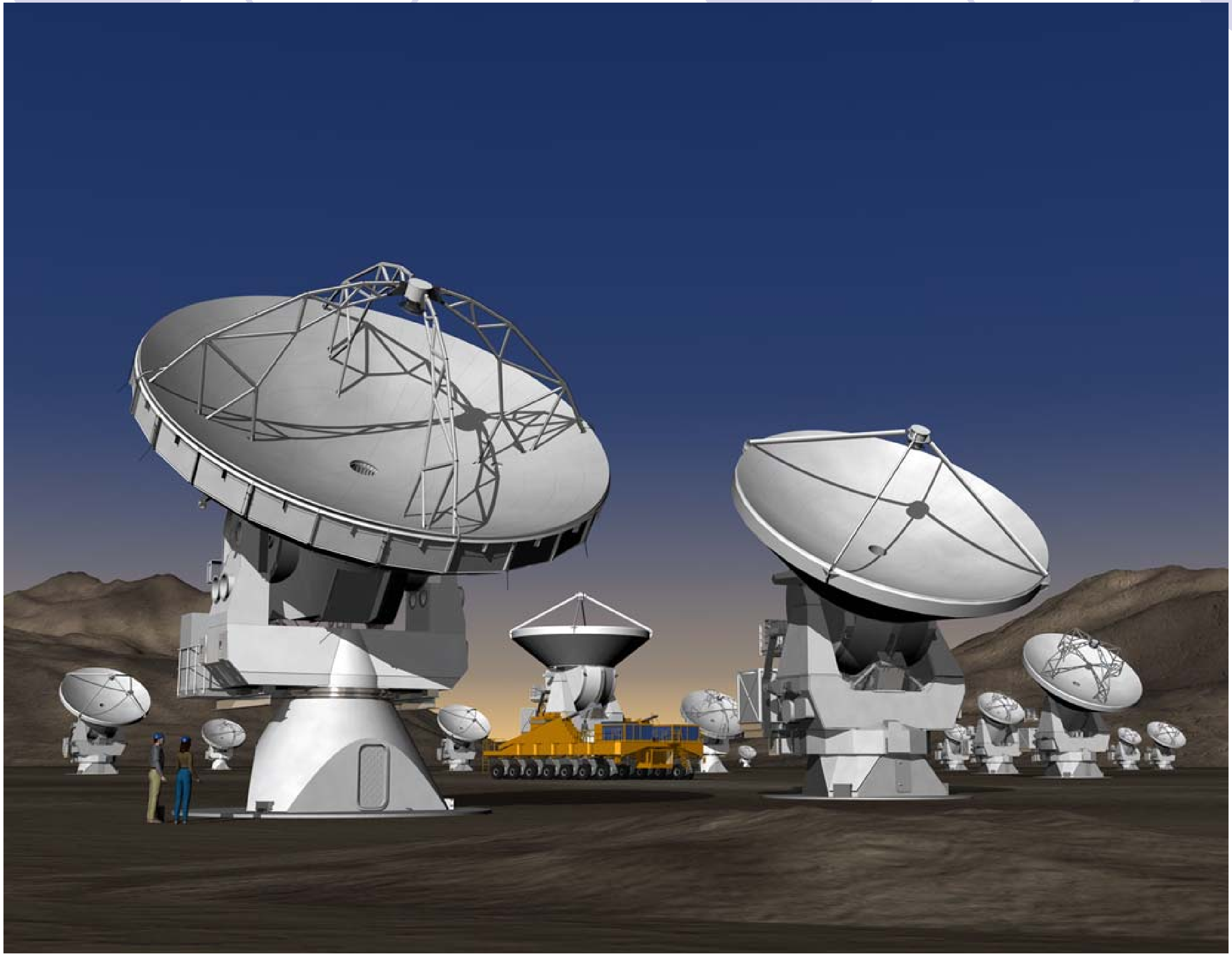
OVRO / BIMA SZE imaging



100 meter offset NRAO Green Bank Telescope



ALMA: 64 -12m telescopes, Atacama, Chile



Atacama Pathfinder EXperiment
APEX

Caltech Submillimeter Observatory

The *Atacama Cosmology Telescope* (ACT)

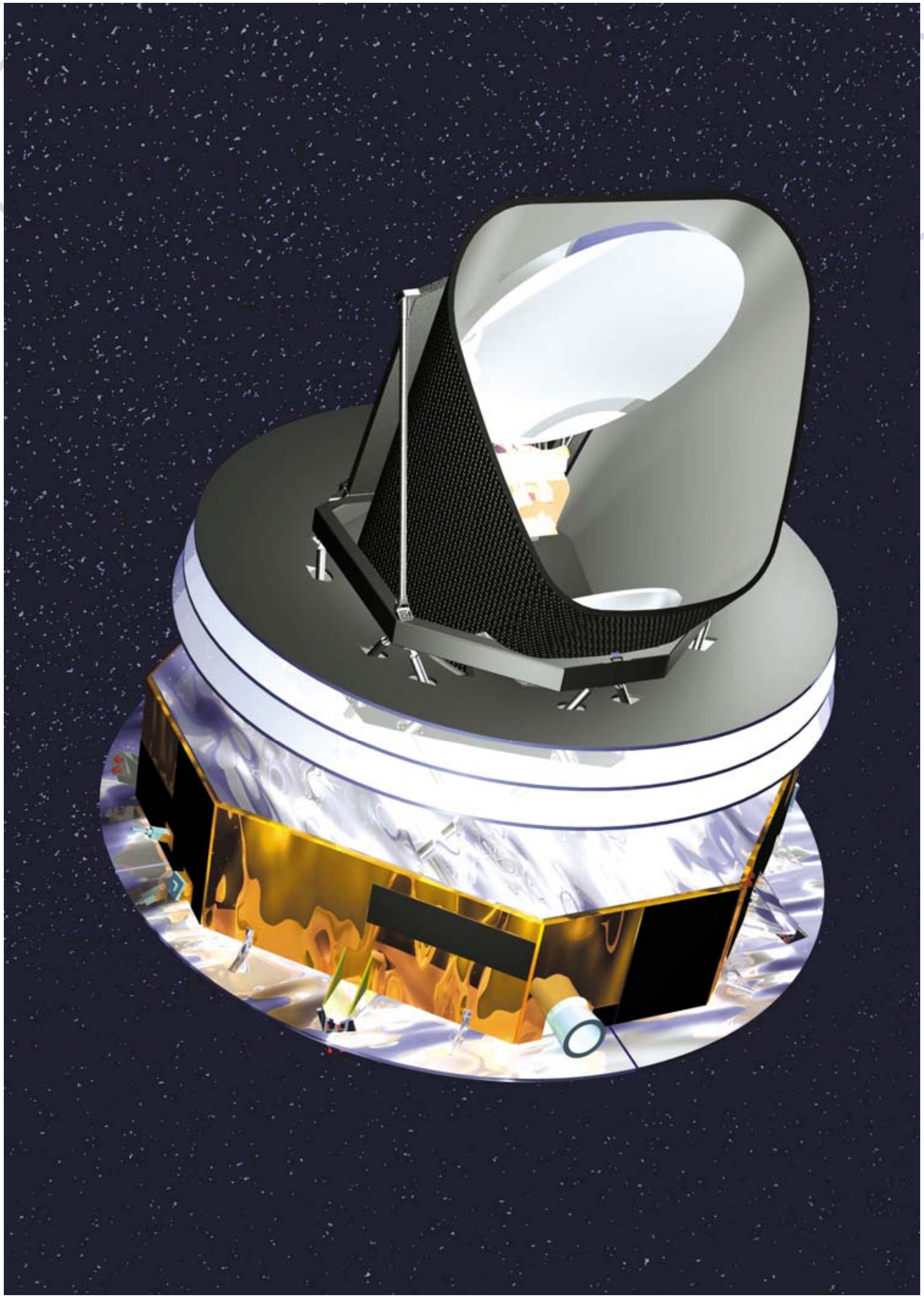
The Arcminute Microkelvin Imager

Sunyaev-Zeldovich Array

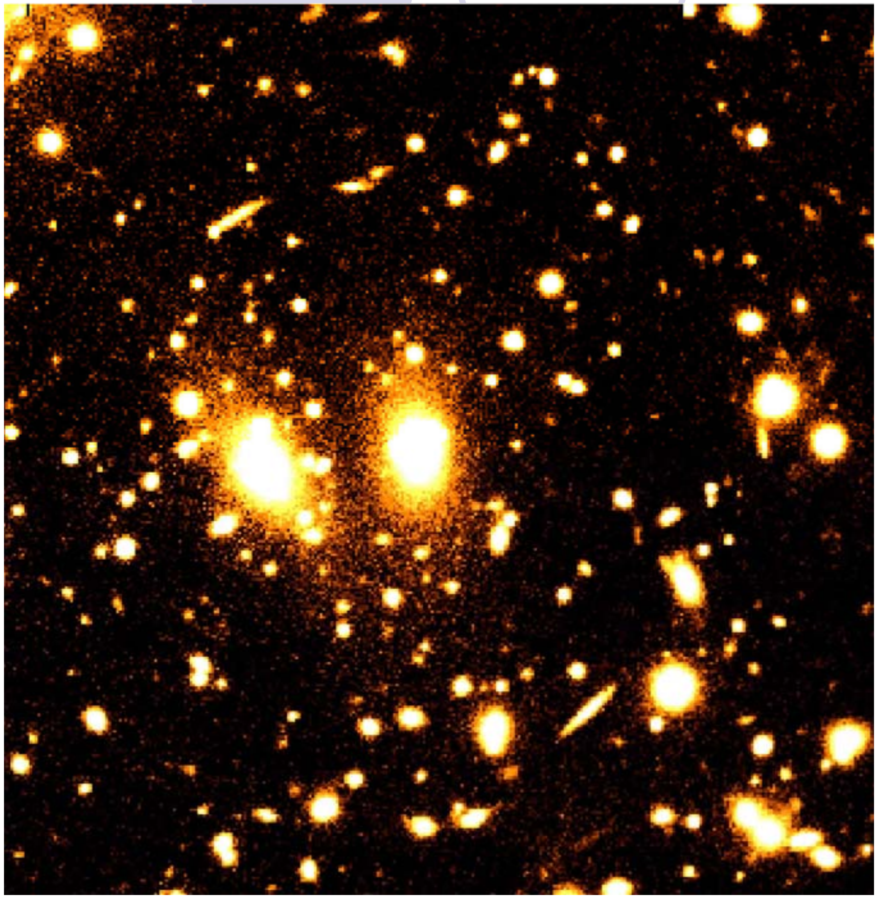
SOUTH POLE TELESCOPE
at Amundsen-Scott South Pole Station

Observation Frequencies

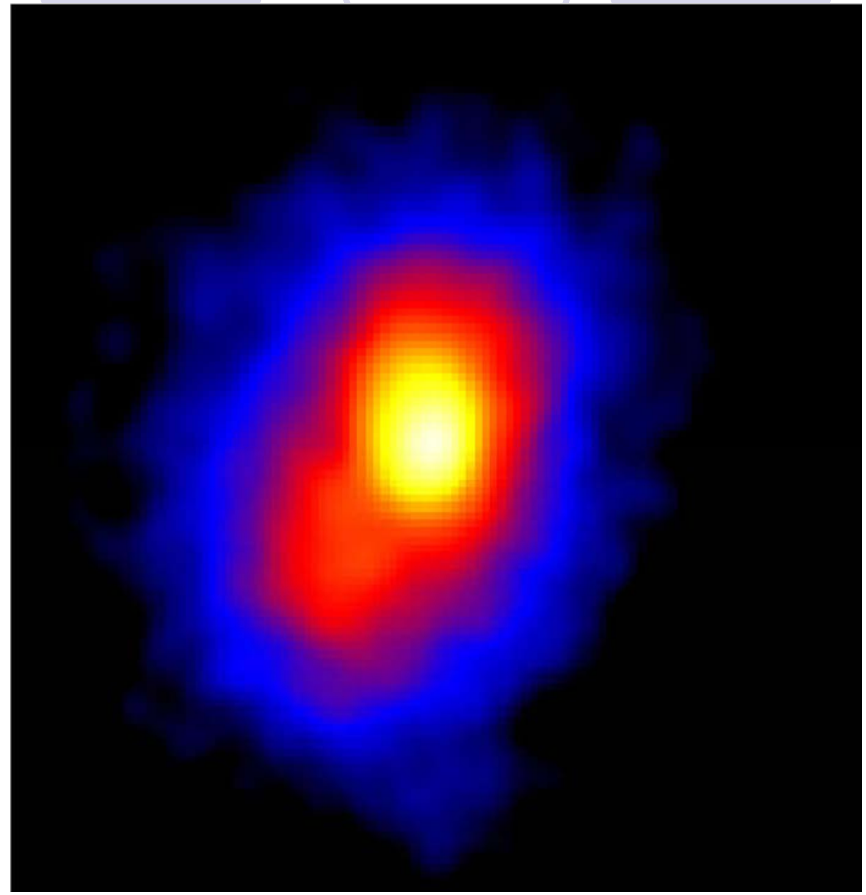
90 GHz
150 GHz
220 GHz
270 GHz



Galaxy Cluster RX J 1347.5-1145 z = 0.451



Subaru Telescope Optical Image



Chandra X-ray Observatory Image

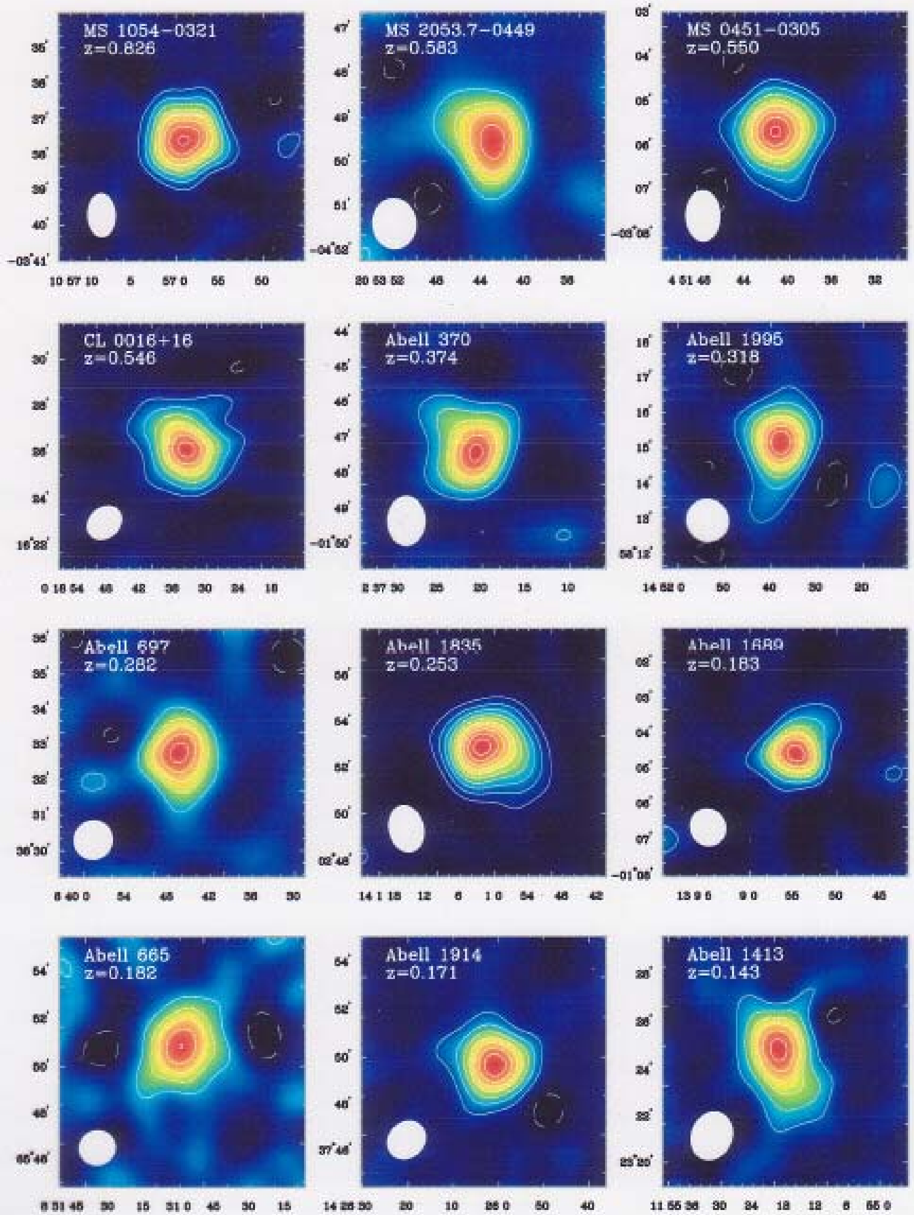
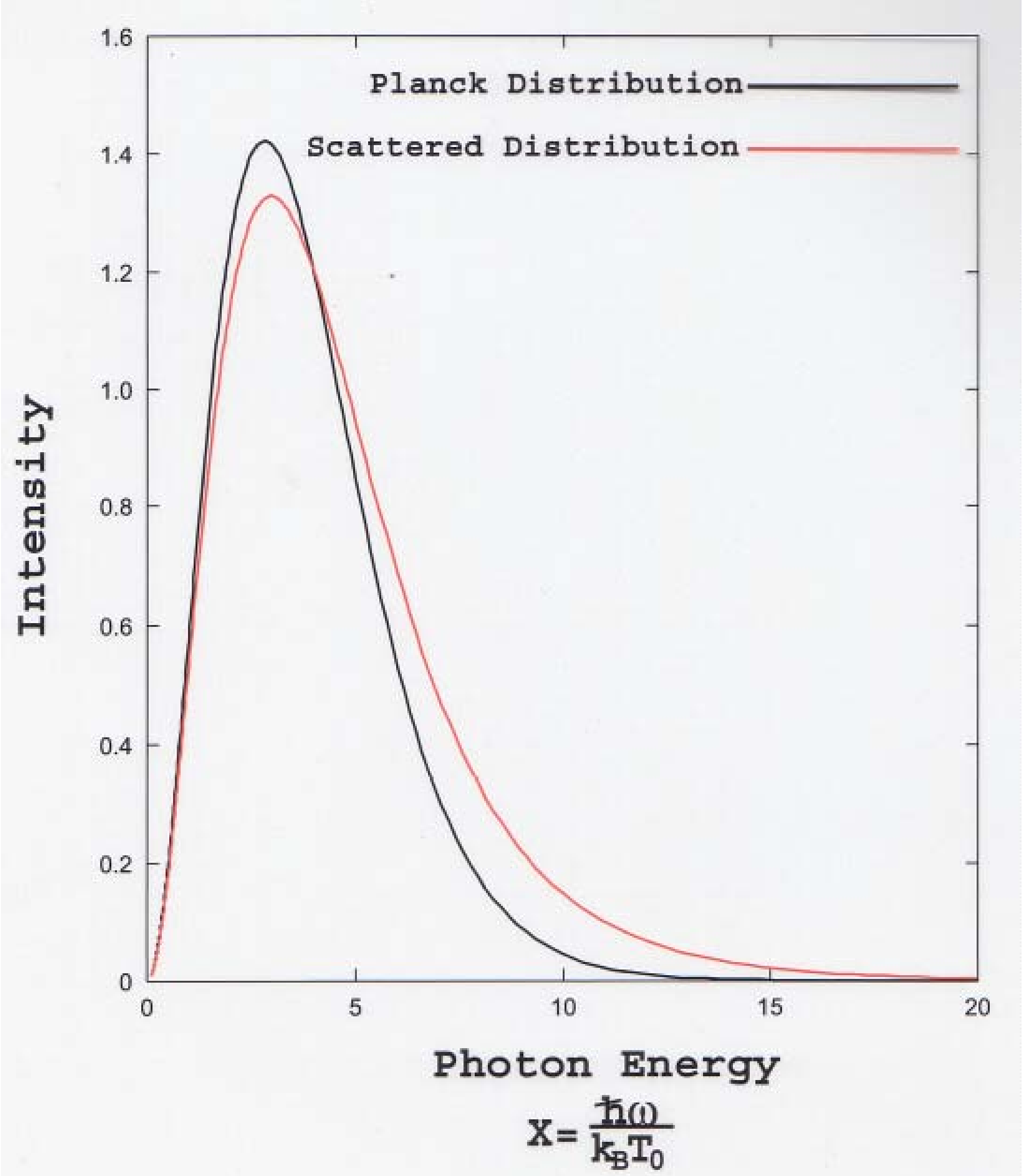


Figure 3: Images of the Sunyaev-Zel'dovich effect toward twelve distant clusters with redshifts spanning 0.83 (top left) to 0.14 (bottom right). The evenly spaced contours are multiples starting at ± 1 of 1.5σ to 3σ depending on the cluster, where σ is the rms noise level in the images. The noise levels range from 15 to 40 μK. The data were taken with the OVRO and BIMA mm-arrays outfitted with low-noise cm-wave receivers. The filled ellipse shown in the bottom left corner of each panel represents the FWHM of the effective resolution used to make these images.



Planck Distribution
Scattered Distribution
Intensity
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
5
10
15
20
Photon Energy
$X = \frac{\hbar\omega}{k_B T_0}$

RELATIVISTIC CORRECTIONS TO THE SZ EFFECT

- RELATIVISTIC THERMAL SZ EFFECT
  Rephaeli 1995
  Stebbins 1997
  Challinor and Lasenby ApJ 1998 May 20
  Itoh, Kohyama, and Nozawa ApJ 1998 July 20
- RELATIVISTIC KINEMATICAL SZ EFFECT
  Nozawa, Itoh, and Kohyama ApJ 1998 November 20
  Sazonov and Sunyaev ApJ 1998 November 20
  Challinor and Lasenby ApJ 1999 January 10
- MULTIPLE SCATTERING CONTRIBUTIONS
  Itoh, Kawana, Nozawa, and Kohyama MNRAS 2001
  Dolgov, Hansen, Pastor, and Semikoz ApJ 2001

GENERALIZED KOMPANEETS EQUATION

- We extend the Kompaneets equation to the relativistic regime.
- We formulate the kinetic equation for the photon distribution function using a covariant formalism (Berestetskii, Lifshitz, and Pitaevskii 1982).
- As a reference system we choose the system that is fixed to the center of mass of the cluster of galaxies (which is fixed to the cosmic microwave background radiation frame).

BOLTZMANN EQUATION FOR THE PHOTON DISTRIBUTION FUNCTION

- Time evolution of the photon distribution function is given by

$$\frac{\partial n(\omega)}{\partial t} = -2\int \frac{d^3p}{(2\pi)^3} d^3p' d^3k' \, W \{n(\omega)[1+n(\omega')]f(E) - n(\omega')[1+n(\omega)]f(E')\} \,, \quad (1)$$

$$W = \frac{(e^2/4\pi)^2 \, \overline{X} \, \delta^4(p+k-p'-k')}{2\omega\omega' E E'} \,,$$

$$\overline{X} = -\left(\frac{\kappa}{\kappa'} + \frac{\kappa'}{\kappa}\right) + 4m^4\left(\frac{1}{\kappa} + \frac{1}{\kappa'}\right)^2 - 4m^2\left(\frac{1}{\kappa} + \frac{1}{\kappa'}\right) \,,$$

$$\kappa = -2(p\cdot k) = -2\omega E\left(1 - \frac{|\vec{p}\,|}{E}\cos\alpha\right) \,,$$

$$\kappa' = 2(p\cdot k') = 2\omega' E\left(1 - \frac{|\vec{p}\,|}{E}\cos\alpha'\right) \,.$$

W :transition probability for the Compton scattering

$p = (E, \boldsymbol{p})$ $p' = (E', \boldsymbol{p}')$:electron four-momenta

$k = (\omega, \boldsymbol{k})$, $k' = (\omega', \boldsymbol{k}')$:photon four-momenta

RELATIVISTIC MAXWELLIAN DISTRIBUTION FOR ELECTRONS

$$f(E) = \left[e^{\{(E-m)-(\mu-m)\}/k_B T_e} + 1\right]^{-1}$$
$$\approx e^{-\{K-(\mu-m)\}/k_B T_e}, \qquad (2)$$

Substituting eq.(2) into eq.(1), we obtain as follows.

$$\frac{\partial n(\omega)}{\partial t} = -2 \int \frac{d^3p}{(2\pi)^3} d^3p' d^3k' \, W \, f(E)$$
$$\left\{[1+n(\omega')]n(\omega) - [1+n(\omega)]n(\omega')e^{\Delta x}\right\}, \qquad (3)$$

where

$$x \equiv \frac{\omega}{k_B T_e},$$
$$\Delta x \equiv \frac{\omega' - \omega}{k_B T_e}.$$

FOKKER-PLANCK EXPANSION

We assume $\Delta x \equiv \frac{\omega' - \omega}{k_B T_e} \ll 1$

(Note $\frac{T_0}{T_e} = \frac{3\mathrm{K}}{10^8\mathrm{K}} = 3 \times 10^{-8}$)

$$\begin{aligned}
\frac{\partial n(\omega)}{\partial t} &= 2\left[\frac{\partial n}{\partial x} + n(1+n)\right] I_1 \\
&+ 2\left[\frac{\partial^2 n}{\partial x^2} + 2(1+n)\frac{\partial n}{\partial x} + n(1+n)\right] I_2 \\
&+ 2\left[\frac{\partial^3 n}{\partial x^3} + 3(1+n)\frac{\partial^2 n}{\partial x^2} + 3(1+n)\frac{\partial n}{\partial x} + n(1+n)\right] I_3 \\
&+ 2\left[\frac{\partial^4 n}{\partial x^4} + 4(1+n)\frac{\partial^3 n}{\partial x^3} + 6(1+n)\frac{\partial^2 n}{\partial x^2} + 4(1+n)\frac{\partial n}{\partial x} + n(1+n)\right] I_4 \\
&+ \cdots ,
\end{aligned}$$

$$I_k \equiv \frac{1}{k!}\int \frac{d^3p}{(2\pi)^3} d^3p' d^3k' \, W \, f(E) \, (\Delta x)^k .$$

EVALUATION OF $I_k \equiv \frac{1}{k!}\int \frac{d^3p}{(2\pi)^3} d^3p' d^3k' W f(E)(\Delta x)^k$

We evaluate the integral by power series expansions of p/m.

Strictly speaking ,the expansions are asymptotic expansions.

Challinor & Lasenby (1998) carried out a calculation up to $O(\theta_e^3)$ terms.

$$\theta_e \equiv \frac{k_B T_e}{mc^2} \approx \frac{1}{50} \quad \text{for} \quad k_B T_e = 10[\mathrm{keV}]$$

We carry out a calculation up to $O(\theta_e^5)$ terms by using the symbolic manipulation computer algebra package Mathematica.

We also carry out a direct numerical integration of the Boltzmann equation.

RESULTS BY MATHEMATICA CALCULATIONS

$$\begin{aligned} I_1 \;=\; & \frac{1}{2}\sigma_T N_e \theta_e x \left\{ 4 - x \,+\, \theta_e \left(10 - \frac{47}{2}x + \frac{21}{5}x^2\right) \right. \\ & + \; \theta_e^2 \left(\frac{15}{2} - \frac{1023}{8}x + \frac{567}{5}x^2 - \frac{147}{10}x^3\right) \\ & + \; \theta_e^3 \left(-\frac{15}{2} - \frac{2505}{8}x + \frac{9891}{10}x^2 - \frac{9551}{20}x^3 + \frac{1616}{35}x^4\right) \\ & \left. + \; \theta_e^4 \left(\frac{135}{32} - \frac{30375}{128}x + \frac{177849}{40}x^2 - \frac{472349}{80}x^3 + \frac{63456}{35}x^4 - \frac{940}{7}x^5\right) \right\} , \end{aligned}$$

$$\begin{aligned} I_2 \;=\; & \frac{1}{2}\sigma_T N_e \theta_e x^2 \left\{ 1 \,+\, \theta_e \left(\frac{47}{2} - \frac{63}{5}x + \frac{7}{10}x^2\right) \right. \\ & + \; \theta_e^2 \left(\frac{1023}{8} - \frac{1302}{5}x + \frac{161}{2}x^2 - \frac{22}{5}x^3\right) \\ & + \; \theta_e^3 \left(\frac{2505}{8} - \frac{10647}{5}x + \frac{38057}{20}x^2 - \frac{2829}{7}x^3 + \frac{682}{35}x^4\right) \\ & \left. + \; \theta_e^4 \left(\frac{30375}{128} - \frac{187173}{20}x + \frac{1701803}{80}x^2 - \frac{44769}{4}x^3 + \frac{61512}{35}x^4 - \frac{510}{7}x^5\right) \right\} , \end{aligned}$$

$$
\begin{aligned}
I_3 &= \frac{1}{2}\sigma_T N_e \theta_e x^3 \left\{ \theta_e \left(\frac{42}{5} - \frac{7}{5}x \right) \right. \\
&+ \theta_e^2 \left(\frac{868}{5} - \frac{658}{5}x + \frac{88}{5}x^2 - \frac{11}{30}x^3 \right) \\
&+ \theta_e^3 \left(\frac{7098}{5} - \frac{14253}{5}x + \frac{8084}{7}x^2 - \frac{3503}{28}x^3 + \frac{64}{21}x^4 \right) \\
&+ \left. \theta_e^4 \left(\frac{62391}{10} - \frac{614727}{20}x + 28193x^2 - \frac{123083}{16}x^3 + \frac{14404}{21}x^4 - \frac{344}{21}x^5 \right) \right\} ,
\end{aligned}
$$

$$
\begin{aligned}
I_4 &= \frac{1}{2}\sigma_T N_e \theta_e x^4 \left\{ \frac{7}{10}\theta_e \right. \\
&+ \theta_e^2 \left(\frac{329}{5} - 22x + \frac{11}{10}x^2 \right) \\
&+ \theta_e^3 \left(\frac{14253}{10} - \frac{9297}{7}x + \frac{7781}{28}x^2 - \frac{320}{21}x^3 + \frac{16}{105}x^4 \right) \\
&+ \left. \theta_e^4 \left(\frac{614727}{40} - \frac{124389}{4}x + \frac{239393}{16}x^2 - \frac{7010}{3}x^3 + \frac{12676}{105}x^4 - \frac{11}{7}x^5 \right) \right\} ,
\end{aligned}
$$

$$I_5 = \frac{1}{2}\sigma_T N_e \theta_e x^5 \left[\theta_e^2 \left(\frac{44}{5} - \frac{11}{10} x \right) + \theta_e^3 \left(\frac{18594}{35} - \frac{36177}{140} x + \frac{192}{7} x^2 - \frac{64}{105} x^3 \right) + \theta_e^4 \left(\frac{124389}{10} - \frac{1067109}{80} x + 3696 x^2 - \frac{5032}{15} x^3 + \frac{66}{7} x^4 - \frac{11}{210} x^5 \right) \right],$$

$$I_6 = \frac{1}{2}\sigma_T N_e \theta_e x^6 \left[\frac{11}{30}\theta_e^2 + \theta_e^3 \left(\frac{12059}{140} - \frac{64}{3} x + \frac{32}{35} x^2 \right) + \theta_e^4 \left(\frac{355703}{80} - \frac{8284}{3} x + \frac{6688}{15} x^2 - 22 x^3 + \frac{11}{42} x^4 \right) \right],$$

$$I_7 = \frac{1}{2}\sigma_T N_e \theta_e x^7 \left[\theta_e^3 \left(\frac{128}{21} - \frac{64}{105} x \right) + \theta_e^4 \left(\frac{16568}{21} - \frac{30064}{105} x + \frac{176}{7} x^2 - \frac{11}{21} x^3 \right) \right],$$

$$I_8 = \frac{1}{2}\sigma_T N_e \theta_e x^8 \left[\frac{16}{105}\theta_e^3 + \theta_e^4 \left(\frac{7516}{105} - \frac{99}{7} x + \frac{11}{21} x^2 \right) \right],$$

$$I_9 = \frac{1}{2}\sigma_T N_e \theta_e x^9 \left[\theta_e^4 \left(\frac{22}{7} - \frac{11}{42} x \right) \right],$$

$$I_{10} = \frac{1}{2}\sigma_T N_e \theta_e x^{10} \left(\frac{11}{210}\theta_e^4 \right),$$

SUNYAEV-ZELDOVICH EFFECT

We assume the initial photon distribution of the cosmic microwave background radiation to be Planckian with temperature

T_0 : $n_0(X) = \dfrac{1}{e^X - 1}$ $\qquad X \equiv \dfrac{\omega}{k_B T_0}$

Then, the fractional distortion of the photon spectrum is

$$\frac{\Delta n(X)}{n_0(X)} = \frac{y\,\theta_e X e^X}{e^X - 1}\left[Y_0 + \theta_e Y_1 + \theta_e^2 Y_2 + \theta_e^3 Y_3 + \theta_e^4 Y_4\right],$$

$$\theta_e \equiv k_B T_e / mc^2,\ X \equiv \omega / k_B T_0,\ y \equiv \sigma_T \int d\ell N_e,$$

$$\begin{aligned}
Y_0 &= -4 + \tilde{X}, \\
Y_1 &= -10 + \frac{47}{2}\tilde{X} - \frac{42}{5}\tilde{X}^2 + \frac{7}{10}\tilde{X}^3 + \tilde{S}^2\left(-\frac{21}{5} + \frac{7}{5}\tilde{X}\right), \\
Y_2 &= -\frac{15}{2} + \frac{1023}{8}\tilde{X} - \frac{868}{5}\tilde{X}^2 + \frac{329}{5}\tilde{X}^3 - \frac{44}{5}\tilde{X}^4 + \frac{11}{30}\tilde{X}^5 \\
&\quad + \tilde{S}^2\left(-\frac{434}{5} + \frac{658}{5}\tilde{X} - \frac{242}{5}\tilde{X}^2 + \frac{143}{30}\tilde{X}^3\right) \\
&\quad + \tilde{S}^4\left(-\frac{44}{5} + \frac{187}{60}\tilde{X}\right),
\end{aligned}$$

$$
\begin{aligned}
Y_3 \;=\; & \frac{15}{2} + \frac{2505}{8}\tilde{X} - \frac{7098}{5}\tilde{X}^2 + \frac{14253}{10}\tilde{X}^3 - \frac{18594}{35}\tilde{X}^4 \\
+ \; & \frac{12059}{140}\tilde{X}^5 - \frac{128}{21}\tilde{X}^6 + \frac{16}{105}\tilde{X}^7 \\
+ \; & \tilde{S}^2\left(-\frac{7098}{10} + \frac{14253}{5}\tilde{X} - \frac{102267}{35}\tilde{X}^2 + \frac{156767}{140}\tilde{X}^3 - \frac{1216}{7}\tilde{X}^4 + \frac{64}{7}\tilde{X}^5\right) \\
+ \; & \tilde{S}^4\left(-\frac{18594}{35} + \frac{205003}{280}\tilde{X} - \frac{1920}{7}\tilde{X}^2 + \frac{1024}{35}\tilde{X}^3\right) \\
+ \; & \tilde{S}^6\left(-\frac{544}{21} + \frac{992}{105}\tilde{X}\right) \, ,
\end{aligned}
$$

$$
\begin{aligned}
Y_4 \;=\;& -\frac{135}{32} + \frac{30375}{128}\tilde{X} - \frac{62391}{10}\tilde{X}^2 + \frac{614727}{40}\tilde{X}^3 - \frac{124389}{10}\tilde{X}^4 \\
+\;& \frac{355703}{80}\tilde{X}^5 - \frac{16568}{21}\tilde{X}^6 + \frac{7516}{105}\tilde{X}^7 - \frac{22}{7}\tilde{X}^8 + \frac{11}{210}\tilde{X}^9 \\
+\;& \tilde{S}^2\left(-\frac{62391}{20} + \frac{614727}{20}\tilde{X} - \frac{1368279}{20}\tilde{X}^2 + \frac{4624139}{80}\tilde{X}^3 - \frac{157396}{7}\tilde{X}^4\right. \\
&\quad\left. + \frac{30064}{7}\tilde{X}^5 - \frac{2717}{7}\tilde{X}^6 + \frac{2761}{210}\tilde{X}^7\right) \\
+\;& \tilde{S}^4\left(-\frac{124389}{10} + \frac{6046951}{160}\tilde{X} - \frac{248520}{7}\tilde{X}^2 + \frac{481024}{35}\tilde{X}^3 - \frac{15972}{7}\tilde{X}^4\right. \\
&\quad\left. + \frac{18689}{140}\tilde{X}^5\right) \\
+\;& \tilde{S}^6\left(-\frac{70414}{21} + \frac{465992}{105}\tilde{X} - \frac{11792}{7}\tilde{X}^2 + \frac{19778}{105}\tilde{X}^3\right) \\
+\;& \tilde{S}^8\left(-\frac{682}{7} + \frac{7601}{210}\tilde{X}\right),
\end{aligned}
$$

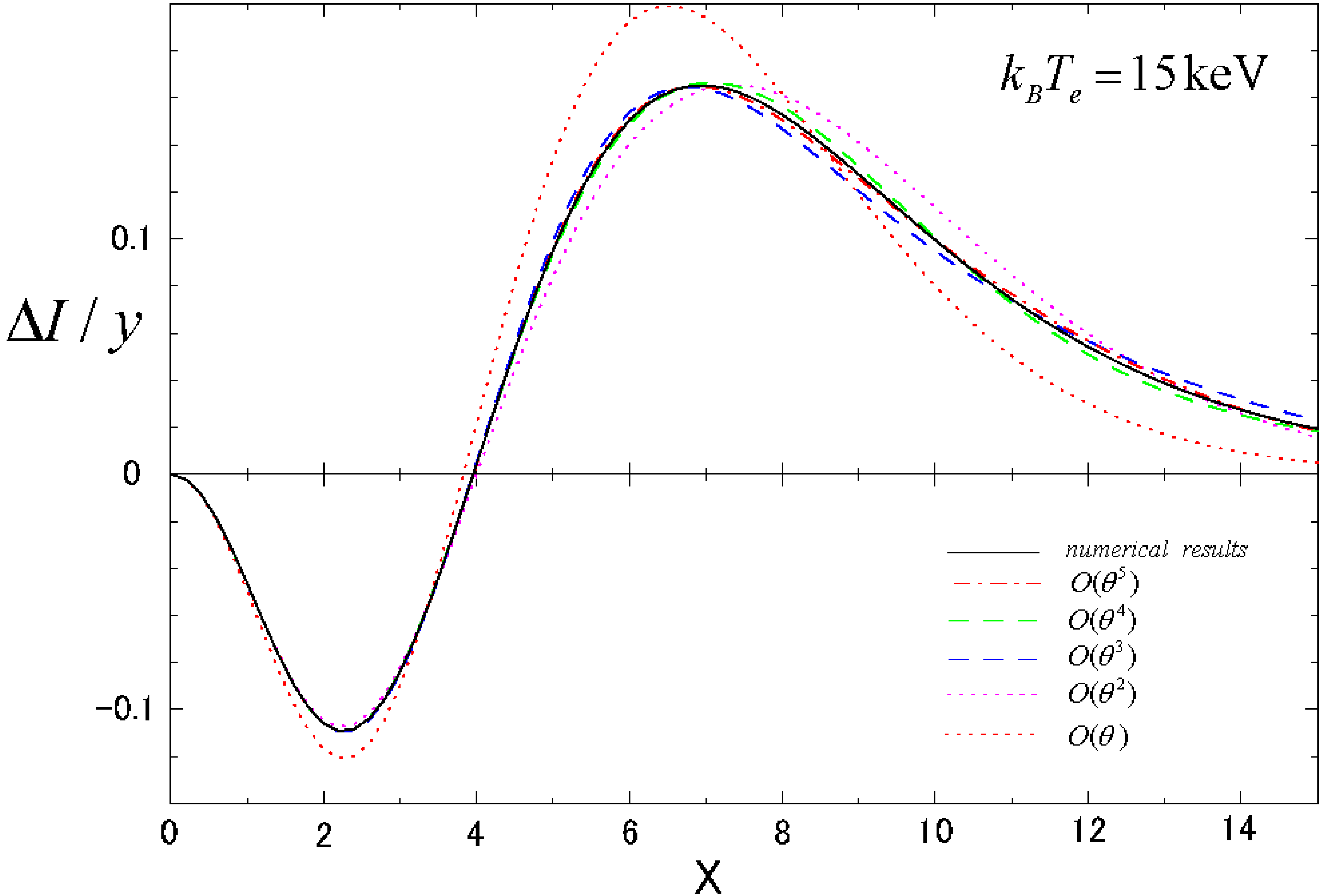
$$
\tilde{X} \;\equiv\; X\coth\left(\frac{X}{2}\right), \quad \tilde{S} \;\equiv\; \frac{X}{\sinh\left(\frac{X}{2}\right)} .
$$

DISTORTION OF THE SPECTRAL INTENSITY

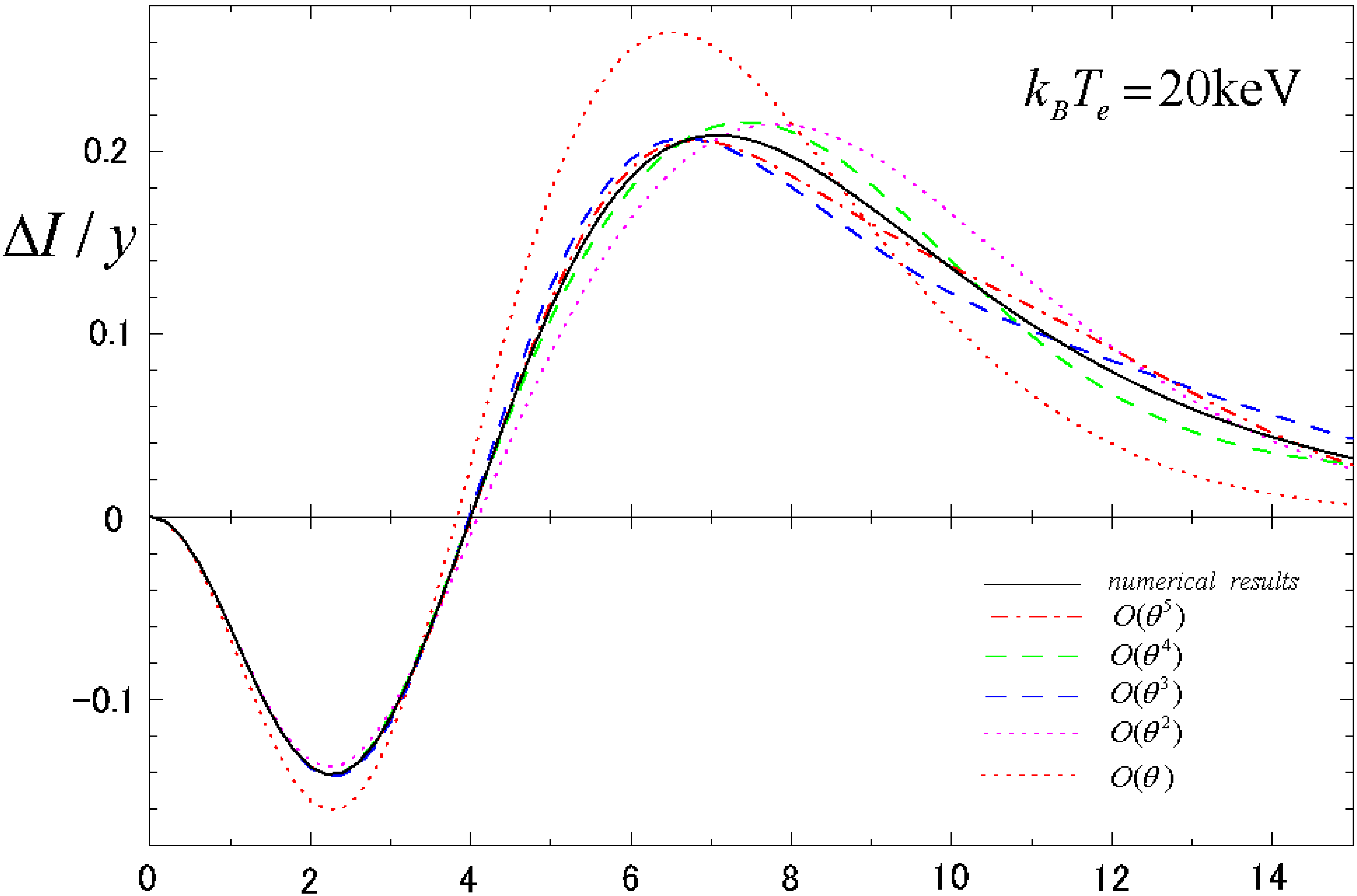
$$\Delta I = \frac{X^3}{e^X - 1} \frac{\Delta n(X)}{n_0(X)} \, .$$

RAYLEIGH-JEANS LIMIT

$$\frac{\Delta n(X)}{n_0(X)} \longrightarrow -2y\,\theta_e\left[1-\frac{17}{10}\theta_e+\frac{123}{40}\theta_e^2-\frac{1989}{280}\theta_e^3+\frac{14403}{640}\theta_e^4\right].$$



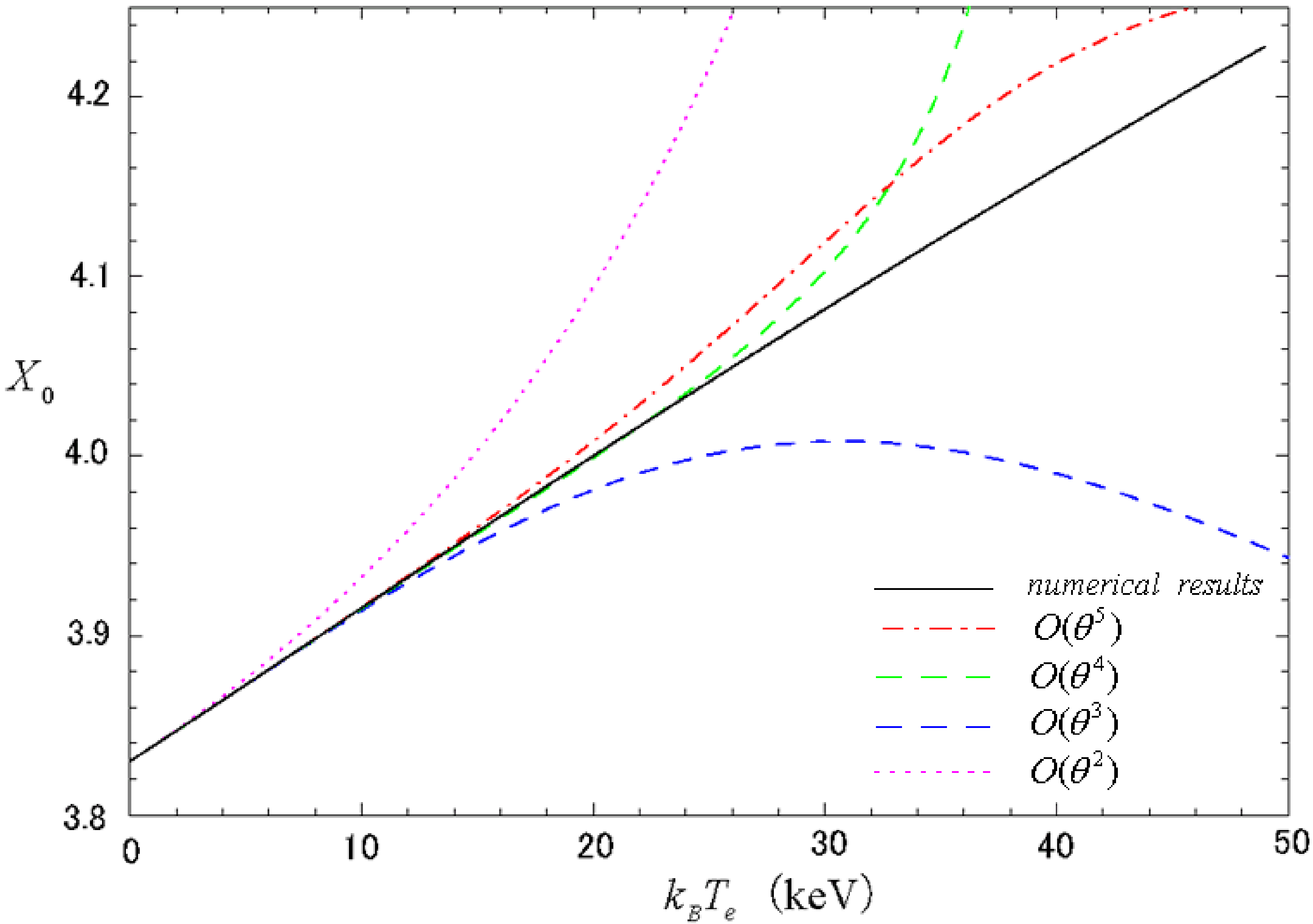
$k_B T_e = 15\,\mathrm{keV}$
$\Delta I / y$
0.1
0
−0.1
0
2
4
6
8
10
12
14
X
numerical results
$O(\theta^5)$
$O(\theta^4)$
$O(\theta^3)$
$O(\theta^2)$
$O(\theta)$



$k_B T_e = 20\mathrm{keV}$
$\Delta I / y$
0.2
0.1
0
−0.1
0
2
4
6
8
10
12
14
numerical results
$O(\theta^5)$
$O(\theta^4)$
$O(\theta^3)$
$O(\theta^2)$
$O(\theta)$

CROSSOVER FRQUENCY

$$X_0 \approx 3.830 \left(1 + 1.1674\theta_e - 0.8533\theta_e^2\right).$$

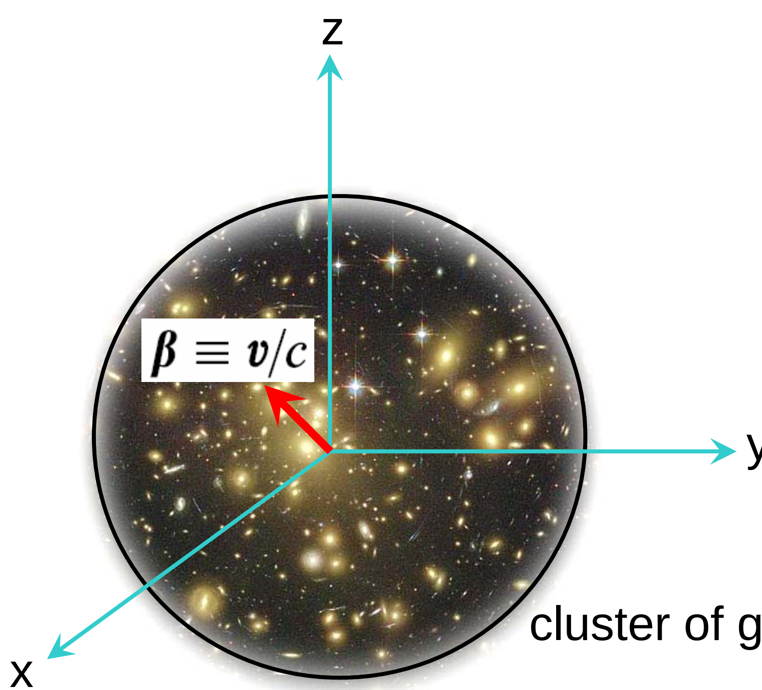


KINEMATICAL S-Z EFFECT

A cluster of galaxies (CG) is moving with a peculiar velocity $\beta \equiv v/c$ with respect to the cosmic microwave background radiation (CMBR).

As a reference system, we choose the system which is fixed to CMBR. We assume that the observer is fixed to the CMBR frame. The Z-axis is fixed to a line connecting the observer and the center of mass of CG.

observer



cluster of galaxies

LORENTZ-BOOSTED KOMPANEETS EQUATION

The electron distribution functions are Fermi-like in the CG frame. They are related to the electron distribution functions in the CMBR frame by

$$f(E) = f_C(E_C) \qquad E_C = \gamma(E - \boldsymbol{\beta} \cdot \boldsymbol{p}) \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$f(E') = f_C(E'_C) \qquad E'_C = \gamma(E' - \boldsymbol{\beta} \cdot \boldsymbol{p}')$$

$$f_C(E_C) = \left(e^{[(E_C - m) - (\mu_C - m)]/k_B T_e} + 1\right)^{-1} \approx e^{-[(E_C - m) - (\mu_C - m)]/k_B T_e}$$

$$x \equiv \frac{\omega}{k_B T_e} \qquad \Delta x \equiv \frac{\omega' - \omega}{k_B T_e}$$

Boltzmann equation in the CMBR frame

$$\frac{\partial n(\omega)}{\partial t} = -2 \int \frac{d^3 p}{(2\pi)^3}\, d^3 p'\, d^3 k'\, W$$

$$\{n(\omega)[1 + n(\omega')]\, f(E) - n(\omega')[1 + n(\omega)]\, f(E')\}$$

KINEMATICAL SZ EFFECT

$$
\begin{aligned}
\frac{\Delta n(X)}{n_0(X)} &= \frac{y\,Xe^X}{e^X-1}\,\theta_e\left[\,Y_0+\theta_e Y_1+\theta_e^2 Y_2+\theta_e^3 Y_3+\theta_e^4 Y_4\,\right] \\
&+ \frac{y\,Xe^X}{e^X-1}\,\beta^2\left[\,B_0+\theta_e B_1+\theta_e^2 B_2+\theta_e^3 B_3\,\right] \\
&+ \frac{y\,Xe^X}{e^X-1}\,\beta\,P_1(\cos\theta_\gamma)\left[\,C_0+\theta_e C_1+\theta_e^2 C_2+\theta_e^3 C_3+\theta_e^4 C_4\,\right] \\
&+ \frac{y\,Xe^X}{e^X-1}\beta^2 P_2(\cos\theta_\gamma)\left[\,D_0+\theta_e D_1+\theta_e^2 D_2+\theta_e^3 D_3\,\right], \\
y &\equiv \sigma_T\int d\ell N_e\,,\quad \theta_e\equiv\frac{k_B T_e}{m_e c^2}\,,\quad \cos\theta_\gamma=\frac{\beta_z}{\beta}\,, \\
P_1(\cos\theta_\gamma) &= \cos\theta_\gamma\,, \\
P_2(\cos\theta_\gamma) &= \frac{1}{2}\left(3\cos^2\theta_\gamma-1\right),
\end{aligned}
$$

θ_γ :angle between the directions of the peculiar velocity of the cluster $\boldsymbol{\beta}$ and the initial photon momentum $\boldsymbol{k}$ which is chosen as the positive Z-axis

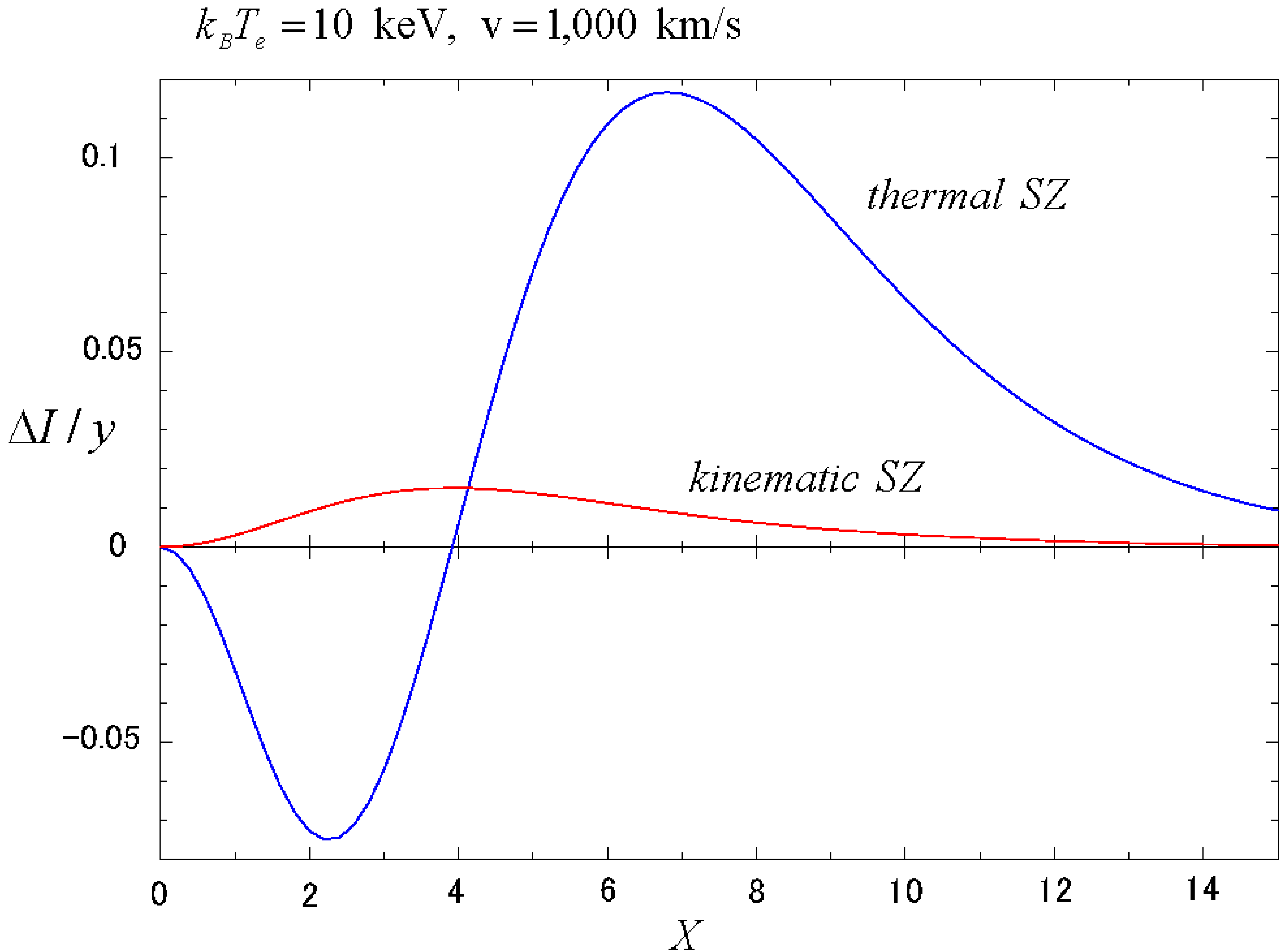
$$
\begin{aligned}
B_0 &= \frac{1}{3}Y_0\,, \\
B_1 &= \frac{5}{6}Y_0 + \frac{2}{3}Y_1\,, \\
B_2 &= \frac{5}{8}Y_0 + \frac{3}{2}Y_1 + Y_2\,, \\
B_3 &= -\frac{5}{8}Y_0 + \frac{5}{4}Y_1 + \frac{5}{2}Y_2 + \frac{4}{3}Y_3\,,
\end{aligned}
$$

$$
\begin{aligned}
C_0 &= 1\,,\\
C_1 &= 10-\frac{47}{5}\tilde{X}+\frac{7}{5}\tilde{X}^2+\frac{7}{10}\tilde{S}^2\,,\\
C_2 &= 25-\frac{1117}{10}\tilde{X}+\frac{847}{10}\tilde{X}^2-\frac{183}{10}\tilde{X}^3+\frac{11}{10}\tilde{X}^4+\tilde{S}^2\left(\frac{847}{20}-\frac{183}{5}\tilde{X}+\frac{121}{20}\tilde{X}^2\right)+\frac{11}{10}\tilde{S}^4\,,\\
C_3 &= \frac{75}{4}-\frac{21873}{40}\tilde{X}+\frac{49161}{40}\tilde{X}^2-\frac{27519}{35}\tilde{X}^3+\frac{6684}{35}\tilde{X}^4-\frac{3917}{210}\tilde{X}^5+\frac{64}{105}\tilde{X}^6\\
&+\tilde{S}^2\left(\frac{49161}{80}-\frac{55038}{35}\tilde{X}+\frac{36762}{35}\tilde{X}^2-\frac{50921}{210}\tilde{X}^3+\frac{608}{35}\tilde{X}^4\right)\\
&+\tilde{S}^4\left(\frac{6684}{35}-\frac{66589}{420}\tilde{X}+\frac{192}{7}\tilde{X}^2\right)+\frac{272}{105}\tilde{S}^6\,,\\
C_4 &= -\frac{75}{4}-\frac{10443}{8}\tilde{X}+\frac{359079}{40}\tilde{X}^2-\frac{938811}{70}\tilde{X}^3+\frac{261714}{35}\tilde{X}^4-\frac{263259}{140}\tilde{X}^5+\frac{4772}{21}\tilde{X}^6-\frac{1336}{105}\tilde{X}^7+\frac{11}{42}\tilde{X}^8\\
&+\tilde{S}^2\left(\frac{359079}{80}-\frac{938811}{35}\tilde{X}+\frac{1439427}{35}\tilde{X}^2-\frac{3422367}{140}\tilde{X}^3+\frac{45334}{7}\tilde{X}^4-\frac{5344}{7}\tilde{X}^5+\frac{2717}{84}\tilde{X}^6\right)\\
&+\tilde{S}^4\left(\frac{261714}{35}-\frac{4475403}{280}\tilde{X}+\frac{71580}{7}\tilde{X}^2-\frac{85504}{35}\tilde{X}^3+\frac{1331}{7}\tilde{X}^4\right)\\
&+\tilde{S}^6\left(\frac{20281}{21}-\frac{82832}{105}\tilde{X}+\frac{2948}{21}\tilde{X}^2\right)+\frac{341}{42}\tilde{S}^8
\end{aligned}
$$

$$
\begin{aligned}
D_0 &= -\frac{2}{3} + \frac{11}{30}\tilde{X}\,, \\
D_1 &= -4 + 12\tilde{X} - 6\tilde{X}^2 + \frac{19}{30}\tilde{X}^3 \;+\; \tilde{S}^2\left(-3 + \frac{19}{15}\tilde{X}\right)\,, \\
D_2 &= -10 + \frac{542}{5}\tilde{X} - \frac{843}{5}\tilde{X}^2 + \frac{10603}{140}\tilde{X}^3 - \frac{409}{35}\tilde{X}^4 + \frac{23}{42}\tilde{X}^5 \\
&+\; \tilde{S}^2\left(-\frac{843}{10} + \frac{10603}{70}\tilde{X} - \frac{4499}{70}\tilde{X}^2 + \frac{299}{42}\tilde{X}^3\right) \\
&+\; \tilde{S}^4\left(-\frac{409}{35} + \frac{391}{84}\tilde{X}\right)\,, \\
D_3 &= -\frac{15}{2} + \frac{4929}{10}\tilde{X} - \frac{39777}{20}\tilde{X}^2 + \frac{1199897}{560}\tilde{X}^3 - \frac{4392}{5}\tilde{X}^4 + \frac{16364}{105}\tilde{X}^5 - \frac{3764}{315}\tilde{X}^6 + \frac{101}{315}\tilde{X}^7 \\
&+\; \tilde{S}^2\left(-\frac{39777}{40} + \frac{1199897}{280}\tilde{X} - \frac{24156}{5}\tilde{X}^2 + \frac{212732}{105}\tilde{X}^3 - \frac{35758}{105}\tilde{X}^4 + \frac{404}{21}\tilde{X}^5\right) \\
&+\; \tilde{S}^4\left(-\frac{4392}{5} + \frac{139094}{105}\tilde{X} - \frac{3764}{7}\tilde{X}^2 + \frac{6464}{105}\tilde{X}^3\right) \\
&+\; \tilde{S}^6\left(-\frac{15997}{315} + \frac{6262}{315}\tilde{X}\right)\,,
\end{aligned}
$$

RAYLEIGH-JEANS LIMIT

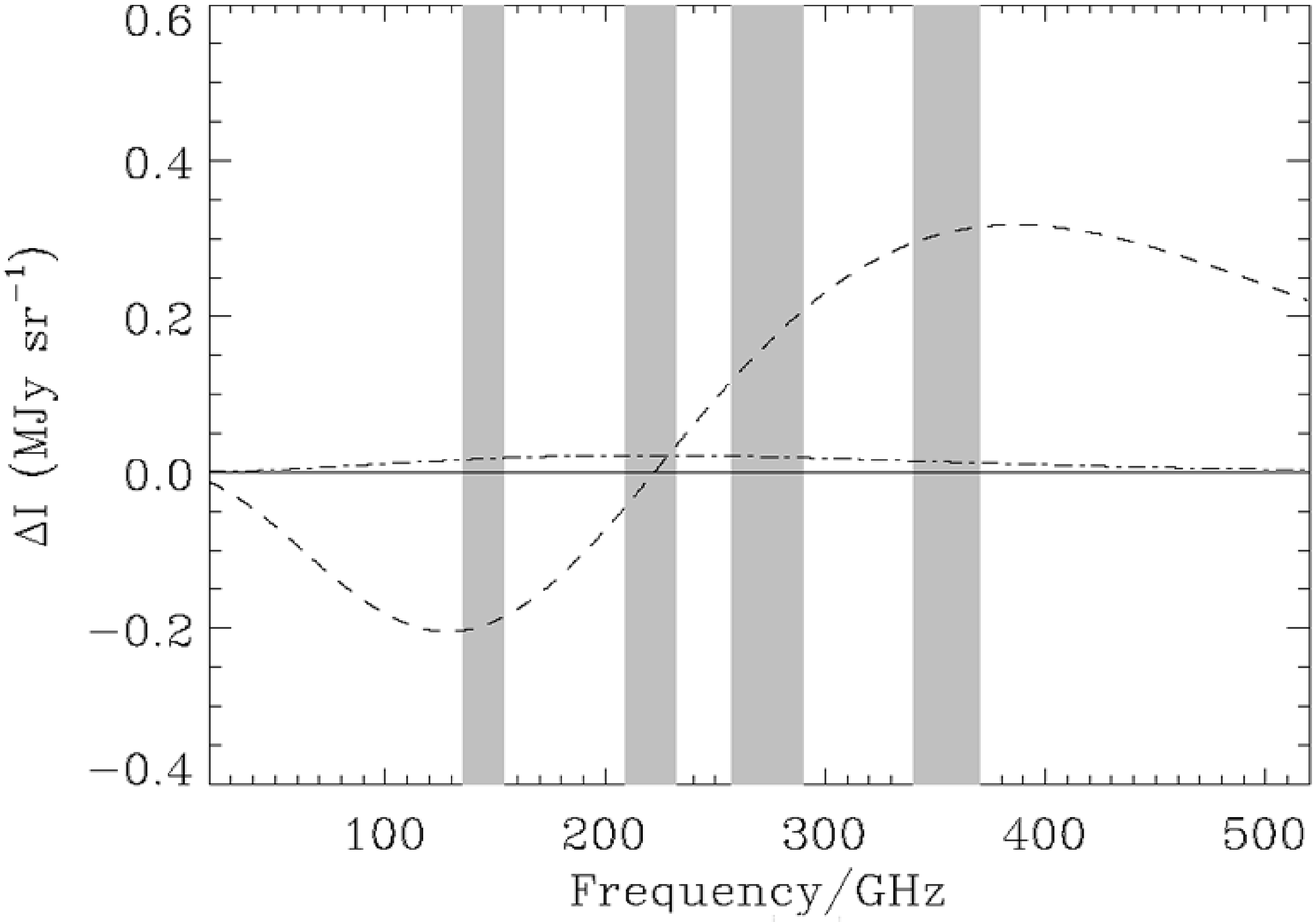
$$\frac{\Delta n(X)}{n_0(X)} \to -2y\,\theta_e \left[1 - \frac{17}{10}\theta_e + \frac{123}{40}\theta_e^2 - \frac{1989}{280}\theta_e^3 + \frac{14403}{640}\theta_e^4\right]$$
$$-2y\,\beta^2 \left[\frac{1}{3} - \frac{3}{10}\,\theta_e + \frac{23}{20}\theta_e^2 - \frac{2539}{560}\theta_e^3\right]$$
$$+\,y\,\beta\,P_1(\cos\theta_\gamma)\left[1 - \frac{2}{5}\theta_e + \frac{13}{5}\theta_e^2 - \frac{1689}{140}\theta_e^3 + \frac{7281}{140}\theta_e^4\right]$$
$$+\,y\,\beta^2\,P_2(\cos\theta_\gamma)\left[\frac{1}{15} - \frac{4}{5}\theta_e + \frac{34}{7}\theta_e^2 - \frac{341}{14}\theta_e^3\right].$$



$k_B T_e = 10$ keV, v = 1,000 km/s
0.1
0.05
0
−0.05
$\Delta I / y$
thermal SZ
kinematic SZ
0
2
4
6
8
10
12
14
X

SuZIE EXPERIMENT

- Benson et al. 2003 ApJ 592, 674 Te=10 keV, v=500 km/s



Herbig, Lawrence, Readhead, & Gulkis 1995

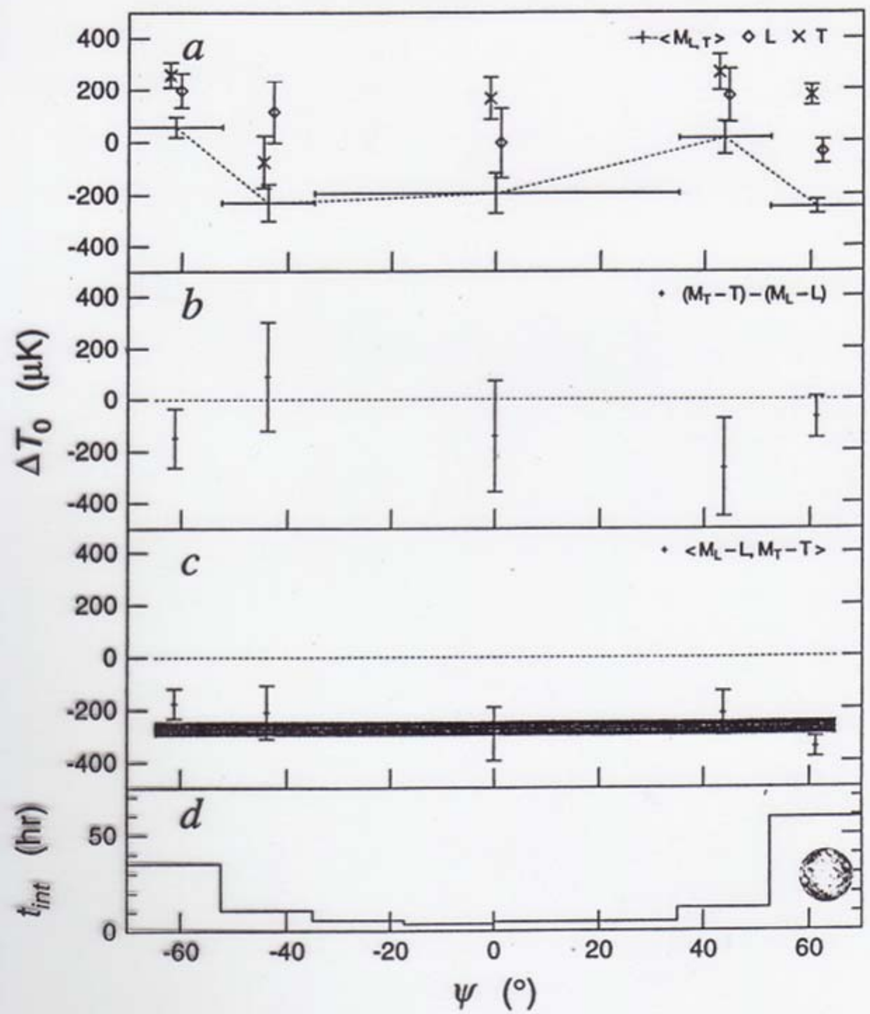


FIG. 1.—Microwave background measurements in the center of the Coma cluster (position C), binned by parallactic angle. The values shown are brightness temperatures on the sky, corrected for the frequency dependence of the SZE. (a) Double-switched data on the main field (M) and the leading (L) and trailing (T) reference fields separately, showing data contaminated by ground pickup. The reference fields lie consistently above the main field, which

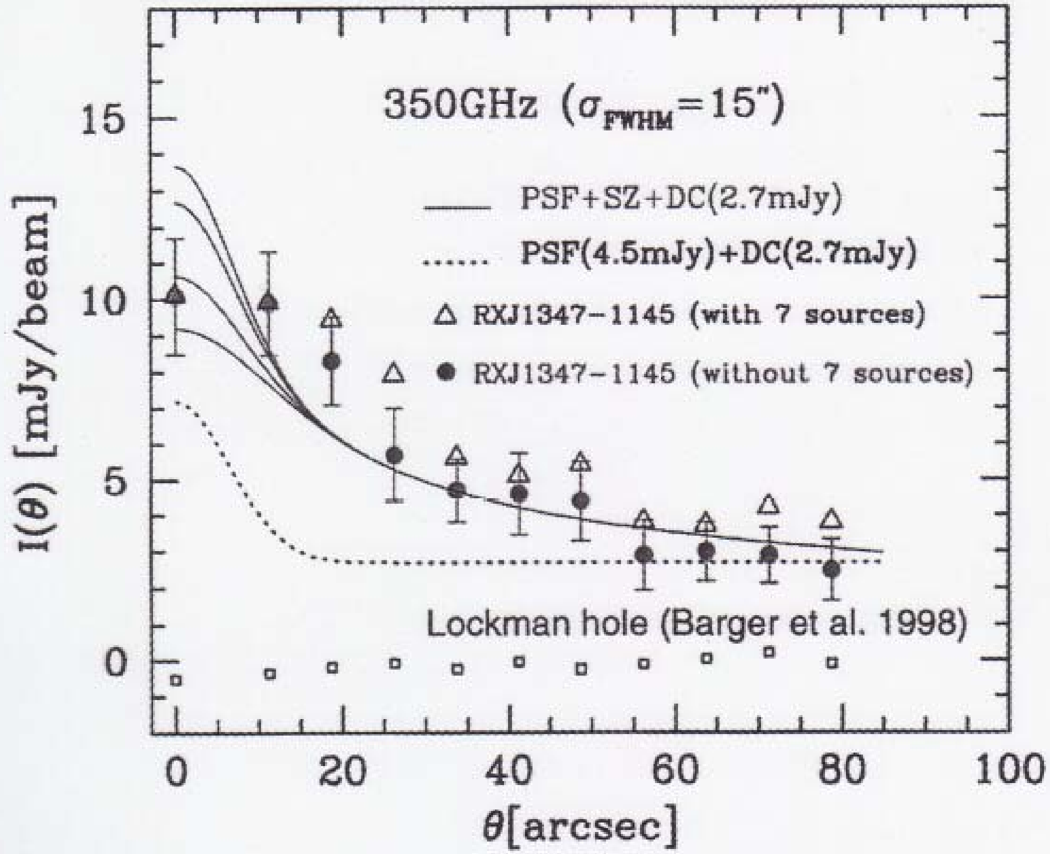


FIG. 3.—Radial intensity profile toward RX J1347 at 350 GHz observed at JCMT/SCUBA. The open triangles (filled circles) indicate our data with (without) the seven spurious sources described in the text. The 1 σ error bars are shown only for the latter. The solid curves plot the SZ profiles from the best-fit parameters in the X-ray observation and the point-source contribution with $F_p = 4.5$ mJy (a conservative 2 σ *upper* limit from eq. [3]), 3.5 mJy (extrapolated from eq. [3]), 1.5 mJy (best fit in Table 1), and 0 mJy (*from top to bottom*). We applied the identical reduction procedure to the Lockman Hole data (Barger et al. 1998), and the results are plotted in small squares for reference (the 1 σ error is smaller than the size of the symbol itself). The dotted curve shows the PSF of a 4.5 mJy source with a 2.7 mJy DC offset.

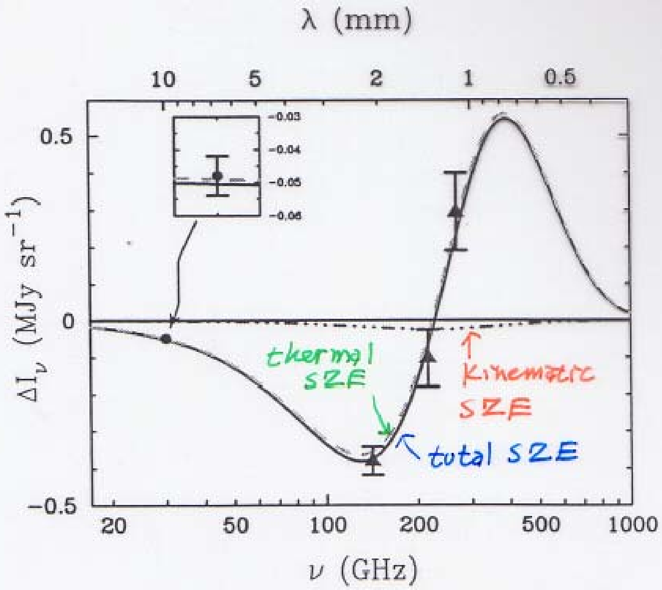


Fig. 2.— SZE spectrum of Abell 2163 (points) and best fit model (lines). The dashed thermal spectrum, dot-dashed the kinetic spectrum, and solid the sum of the two; Su points appear as triangles and OVRO & BIMA as a circle. The insert shows the 1 cm its error bar.

La Roque, Carlstrom, Reese, Holder, Holzapfel, Joy, and Grego 2002

kinematic SZE $v_p = 410^{+1030}_{-850}{}^{+460}_{-440}$ km s^{-1} (statistical & systematic)

Table 1. Measurements of SZE in Abell 2163

λ (mm)	Instrument	Measured ΔI_ν (MJy sr^{-1})	Dust-corrected ΔI_ν (MJy sr^{-1})
10.0	OVRO & BIMA	-0.048 ± 0.006	...
2.1	Diabolo	-0.545 ± 0.22	...
2.1	SuZIE	-0.381 ± 0.037	-0.380 ± 0.037
1.4	SuZIE	-0.106 ± 0.077	-0.103 ± 0.077
1.1	SuZIE	0.287 ± 0.105	0.295 ± 0.105

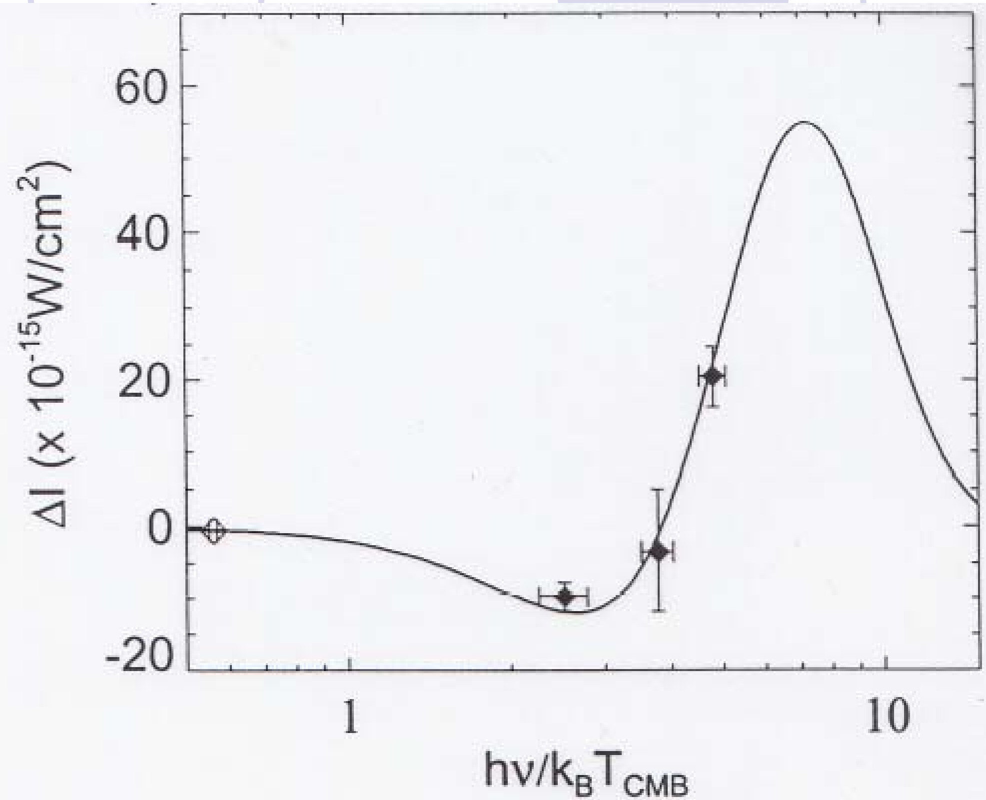
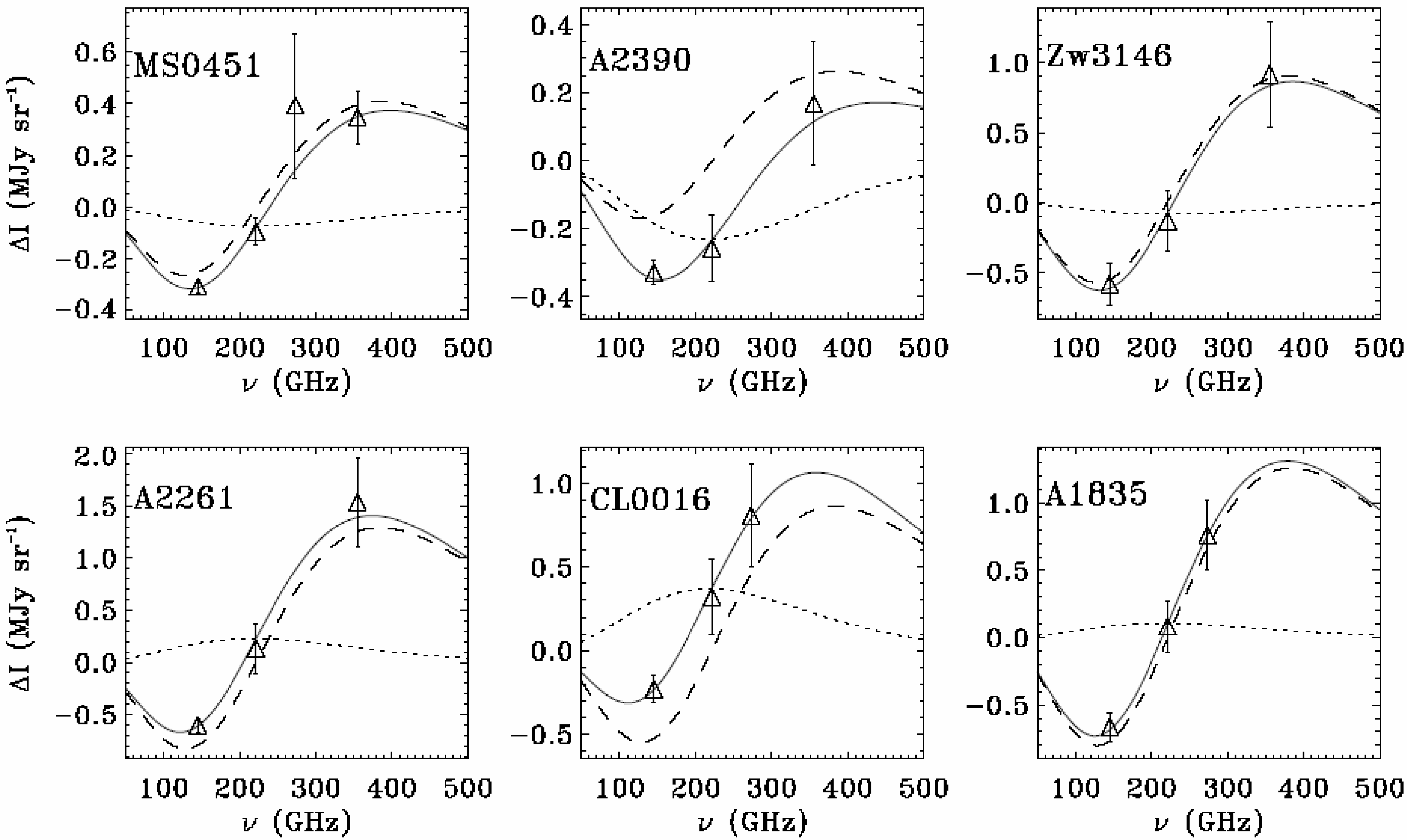


FIG. 2.S-Z spectrum of the Coma cluster. The solid line shows the best fit spectrum (taking isothermal gas with $kT =$ 8.2 keV) to the combined MITO and OVRO (Herbig *et al.* 1995) measurements, corresponding to $\tau \simeq (4.2 \pm 0.7) \times 10^{-3}$.

SUZIE II Results (Benson et al. 2003)



MS0451

$$v = +800 \,^{+1525}_{-1125} \quad \mathrm{km\,s^{-1}}$$

A2390

$$v = +1900 \,^{+6225}_{-2650} \quad \mathrm{km\,s^{-1}}$$

Zw3146

$$v = -400 \,^{+3700}_{-1925} \quad \mathrm{km\,s^{-1}}$$

A2261

$$v = -1575 \,^{+1500}_{-975} \quad \mathrm{km\,s^{-1}}$$

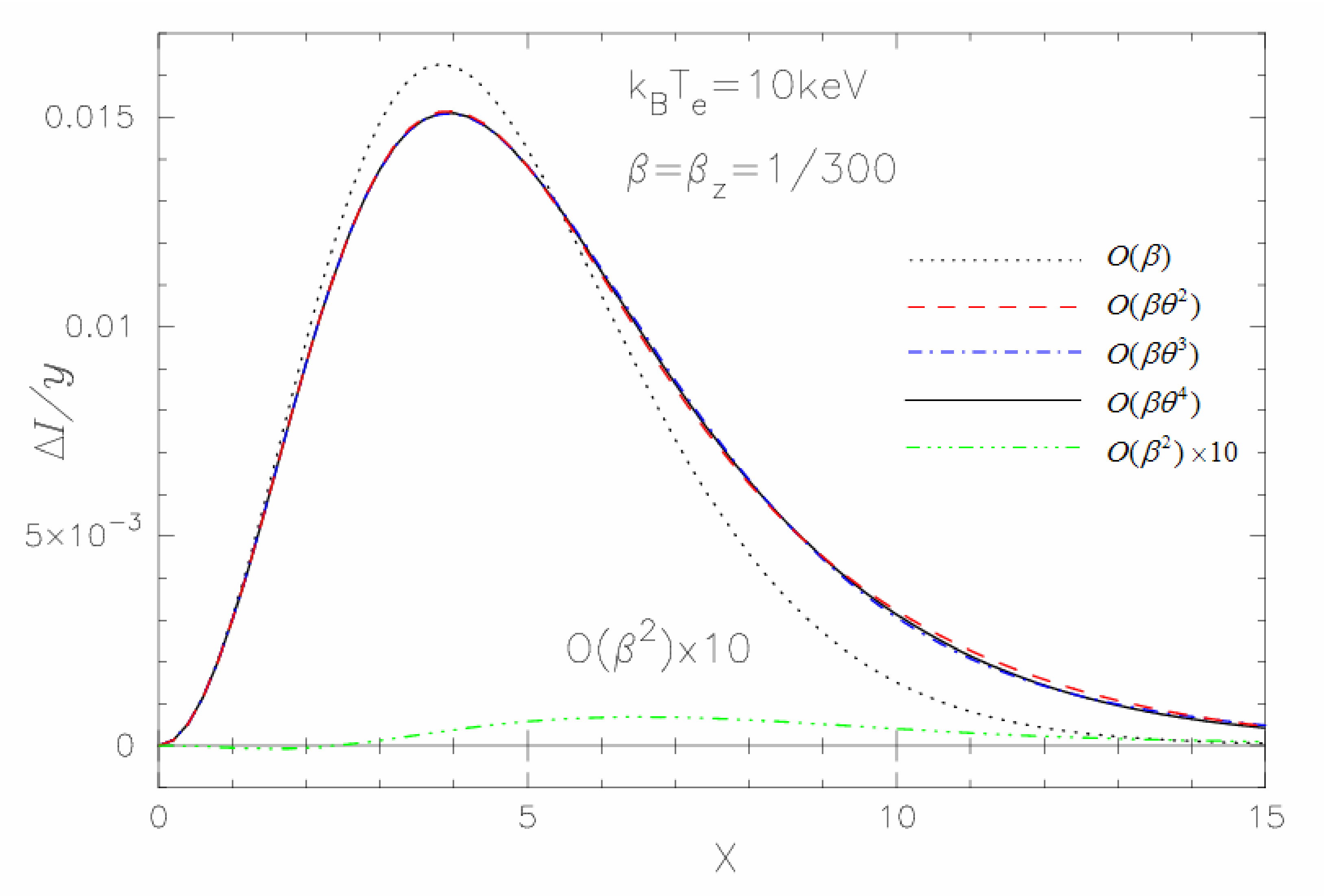
CL0016

$$v = -4100 \,^{+2650}_{-1625} \quad \mathrm{km\,s^{-1}}$$

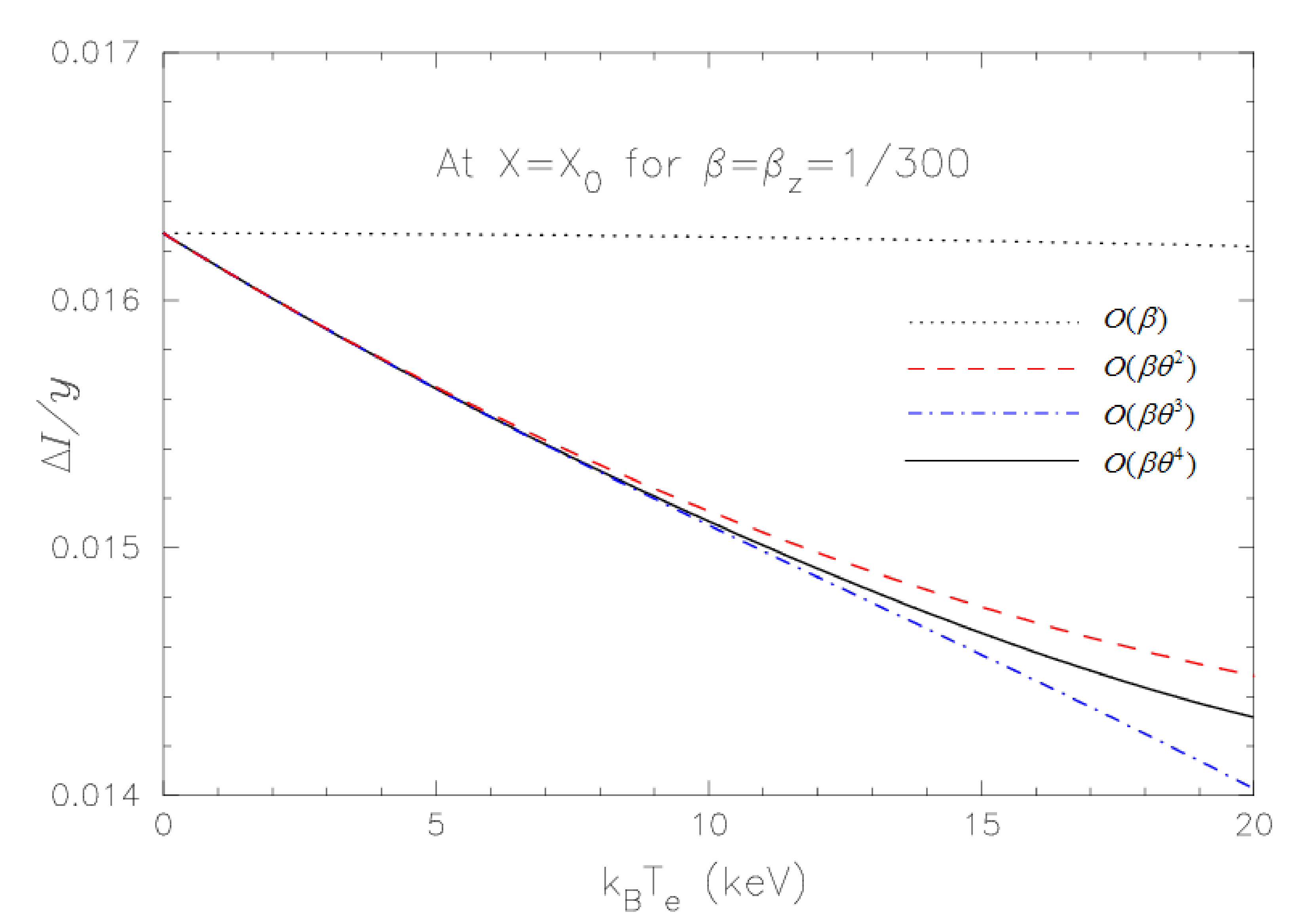
A1835

$$v = -175 \,^{+1675}_{-1275} \quad \mathrm{km\,s^{-1}}$$

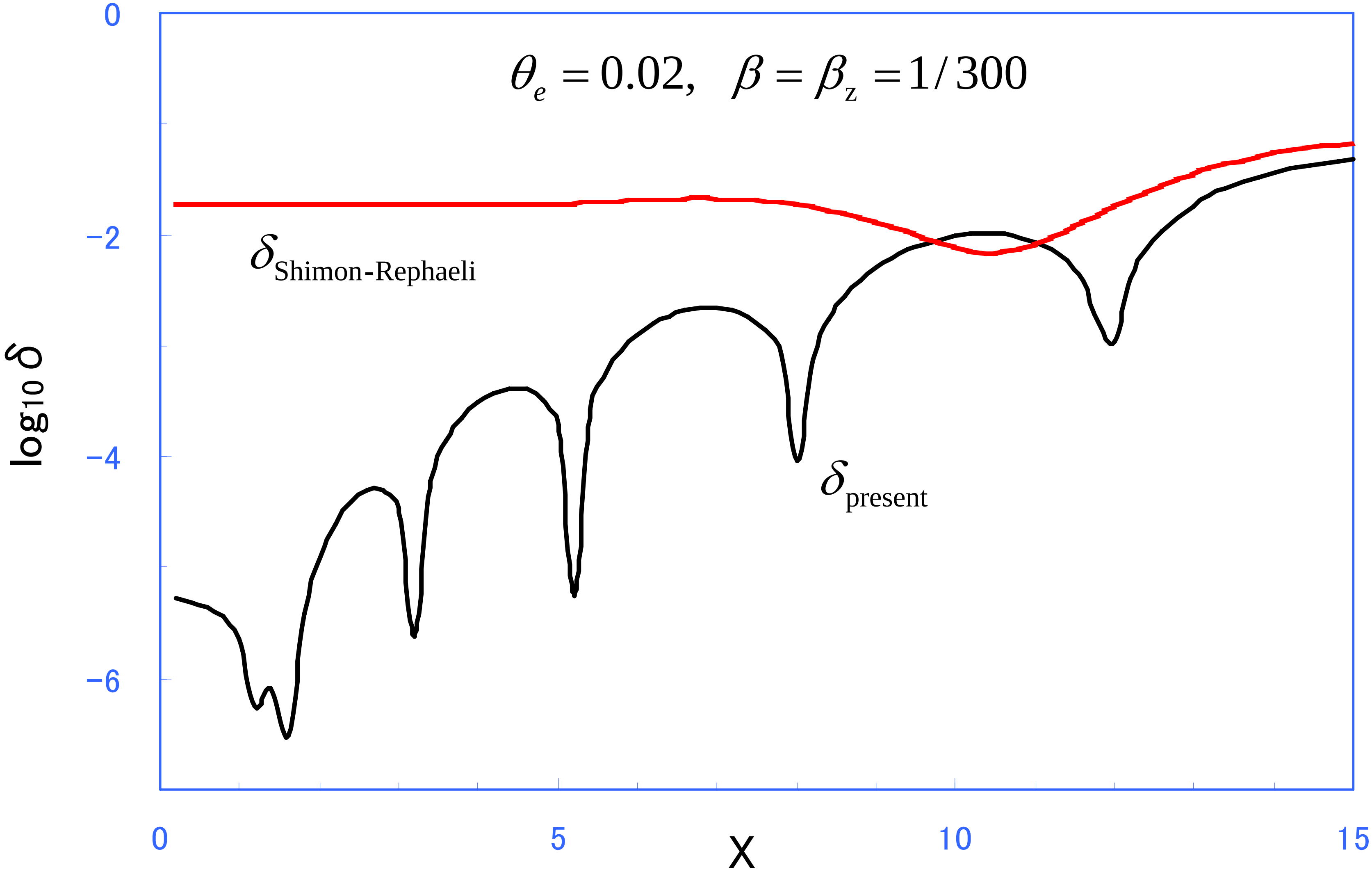
VARIOUS ORDERS OF RELATIVISTIC CORRECTIONS



AT CROSSOVER FRQUENCY



COMPARISON WITH NUMERICAL INTEGRATION



$$\delta_{\text{present}} \equiv \left| \frac{(\Delta I)_{\text{present analytic}} - (\Delta I)_{\text{present numerical}}}{(\Delta I)_{\text{present numerical}}} \right|, \quad \delta_{\text{SR}} \equiv \left| \frac{(\Delta I)_{\text{SR analytic}} - (\Delta I)_{\text{present numerical}}}{(\Delta I)_{\text{present numerical}}} \right|.$$

MULTIPLE SCATTERING CONTRIBUTIONS

- Itoh,Kawana, Nozawa, Kohyama (2001) have calculated the double scattering contributions using the relativistic expansion up to $(\mathrm{T_e/mc^2})^6$.
- Dolgov, Hansen, Pastor, and Semikoz (2001) have carried out a Monte Carlo calculation. Their result shows an excellent agreement with Itoh et al (2001) for the case of small optical depth.

MULTIPLE SCATTERING CONTRIBUTIONS

$$\frac{\Delta n(X)}{n_0(X)} = \frac{y\theta_e X e^X}{e^X - 1}(Y_0 + \theta_e Y_1 + \theta_e^2 Y_2 + \theta_e^3 Y_3 + \theta_e^4 Y_4 + \theta_e^5 Y_5 + \theta_e^6 Y_6)$$
$$+ \frac{1}{2}\frac{y^2\theta_e^2 X e^X}{e^X - 1}(Z_0 + \theta_e Z_1 + \theta_e^2 Z_2 + \theta_e^3 Z_3 + \theta_e^4 Z_4 + \theta_e^5 Z_5 + \theta_e^6 Z_6)$$

$$Z_0 = -16 + 34\tilde{X} - 12\tilde{X}^2 + \tilde{X}^3 + \tilde{S}^2(-6 + 2\tilde{X}),$$

$$Z_1 = -80 + 590\tilde{X} - \frac{3492}{5}\tilde{X}^2 + \frac{1271}{5}\tilde{X}^3 - \frac{168}{5}\tilde{X}^4 + \frac{7}{5}\tilde{X}^5$$
$$+ \tilde{S}^2\left(-\frac{1746}{5} + \frac{2542}{5}\tilde{X} - \frac{924}{5}\tilde{X}^2 + \frac{91}{5}\tilde{X}^3\right) + \tilde{S}^4\left(-\frac{168}{5} + \frac{119}{10}\tilde{X}\right)$$

$$
\begin{aligned}
Z_2 = {} & -160 + 4792\tilde{X} - \frac{357144}{25}\tilde{X}^2 + \frac{312912}{25}\tilde{X}^3 - \frac{110196}{25}\tilde{X}^4 + \frac{34873}{50}\tilde{X}^5 - \frac{734}{15}\tilde{X}^6 + \frac{367}{300}\tilde{X}^7 \\
& + \tilde{S}^2\left(-\frac{178572}{25} + \frac{625824}{25}\tilde{X} - \frac{606078}{25}\tilde{X}^2 + \frac{453349}{50}\tilde{X}^3 - \frac{20919}{15}\tilde{X}^4 + \frac{367}{5}\tilde{X}^5\right) \\
& + \tilde{S}^4\left(-\frac{110196}{25} + \frac{592841}{100}\tilde{X} - 2202\tilde{X}^2 + \frac{5872}{25}\tilde{X}^3\right) + \tilde{S}^6\left(-\frac{6239}{30} + \frac{11377}{150}\tilde{X}\right),
\end{aligned}
$$

$$
\begin{aligned}
Z_3 = {} & -90 + \frac{96651}{4}\tilde{X} - \frac{8659449}{50}\tilde{X}^2 + \frac{62384943}{200}\tilde{X}^3 - \frac{38586081}{175}\tilde{X}^4 + \frac{103117227}{1400}\tilde{X}^5 - \frac{1325008}{105}\tilde{X}^6 + \frac{590831}{525}\tilde{X}^7 - \frac{1718}{35}\tilde{X}^8 \\
& + \frac{859}{1050}\tilde{X}^9 + \tilde{S}^2\left(-\frac{8659449}{100} + \frac{62384943}{100}\tilde{X} - \frac{424446891}{350}\tilde{X}^2 + \frac{1340523951}{1400}\tilde{X}^3 - \frac{12587576}{35}\tilde{X}^4 + \frac{2363324}{35}\tilde{X}^5 - \frac{212173}{35}\tilde{X}^6\right. \\
& \left. + \frac{215609}{1050}\tilde{X}^7\right) + \tilde{S}^4\left(-\frac{38586081}{175} + \frac{1752992859}{2800}\tilde{X} - \frac{3975024}{7}\tilde{X}^2 + \frac{37813184}{175}\tilde{X}^3 - \frac{1247268}{35}\tilde{X}^4 + \frac{1459441}{700}\tilde{X}^5\right) \\
& + \tilde{S}^6\left(-\frac{5631284}{105} + \frac{36631522}{525}\tilde{X} - \frac{920848}{35}\tilde{X}^2 + \frac{1544482}{525}\tilde{X}^3\right) + \tilde{S}^8\left(-\frac{53258}{35} + \frac{593569}{1050}\tilde{X}\right),
\end{aligned}
$$

$$
\begin{aligned}
Z_4 = {} & 60 + 82497\tilde{X} - \frac{36883086}{25}\tilde{X}^2 + \frac{129233103}{25}\tilde{X}^3 - \frac{1154992263}{175}\tilde{X}^4 + \frac{5504779501}{1400}\tilde{X}^5 - \frac{129898756}{105}\tilde{X}^6 + \frac{114929504}{525}\tilde{X}^7 \\
& - \frac{2337809}{105}\tilde{X}^8 + \frac{2681837}{2100}\tilde{X}^9 - \frac{2851}{75}\tilde{X}^{10} + \frac{2851}{6300}\tilde{X}^{11} + \tilde{S}^2\left(-\frac{18441543}{25} + \frac{258466206}{25}\tilde{X} - \frac{12704914893}{350}\tilde{X}^2 \right. \\
& \left. + \frac{71562133513}{1400}\tilde{X}^3 - \frac{1234038182}{35}\tilde{X}^4 + \frac{459718016}{35}\tilde{X}^5 - \frac{577438823}{210}\tilde{X}^6 + \frac{673141087}{2100}\tilde{X}^7 - \frac{2888063}{150}\tilde{X}^8 + \frac{1451159}{3150}\tilde{X}^9 \right) \\
& + \tilde{S}^4\left(-\frac{1154992263}{175} + \frac{93581251517}{2800}\tilde{X} - \frac{389696268}{7}\tilde{X}^2 + \frac{7355488256}{175}\tilde{X}^3 - \frac{565749778}{35}\tilde{X}^4 + \frac{4556441063}{1400}\tilde{X}^5 \right. \\
& \left. - \frac{24301924}{75}\tilde{X}^6 + \frac{39340949}{3150}\tilde{X}^7 \right) + \tilde{S}^6\left(-\frac{552069713}{105} + \frac{7125629248}{525}\tilde{X} - \frac{1253065624}{105}\tilde{X}^2 + \frac{2410971463}{525}\tilde{X}^3 \right. \\
& \left. - \frac{236692871}{300}\tilde{X}^4 + \frac{309598643}{6300}\tilde{X}^5 \right) + \tilde{S}^8\left(-\frac{72472079}{105} + \frac{1853149367}{2100}\tilde{X} - \frac{25228499}{75}\tilde{X}^2 + \frac{488977861}{12600}\tilde{X}^3 \right) \\
& + \tilde{S}^{10}\left(-\frac{1970041}{150} + \frac{15569311}{3150}\tilde{X} \right),
\end{aligned}
$$

$$
\begin{aligned}
Z_5 = & -\frac{135}{8} + \frac{12368565}{64}\tilde{X} - \frac{1523246139}{160}\tilde{X}^2 + \frac{41024053941}{640}\tilde{X}^3 - \frac{78913341669}{560}\tilde{X}^4 + \frac{124274226315}{896}\tilde{X}^5 - \frac{2505515368}{35}\tilde{X}^6 \\
& + \frac{29643451897}{1400}\tilde{X}^7 - \frac{105635617}{28}\tilde{X}^8 + \frac{10436409287}{25200}\tilde{X}^9 - \frac{6284921}{225}\tilde{X}^{10} + \frac{2347649}{2100}\tilde{X}^{11} - \frac{16312}{675}\tilde{X}^{12} + \frac{2039}{9450}\tilde{X}^{13} \\
& + \tilde{S}^2\left(-\frac{1523246139}{320} + \frac{41024053941}{320}\tilde{X} - \frac{868046758359}{1120}\tilde{X}^2 + \frac{1615564942095}{896}\tilde{X}^3 - \frac{71407187988}{35}\tilde{X}^4 + \frac{88930355691}{70}\tilde{X}^5 \right. \\
& \left. - \frac{26091997399}{56}\tilde{X}^6 + \frac{2619538731037}{25200}\tilde{X}^7 - \frac{6366624973}{450}\tilde{X}^8 + \frac{1194953341}{1050}\tilde{X}^9 - \frac{11100316}{225}\tilde{X}^{10} + \frac{2779157}{3150}\tilde{X}^{11} \right) \\
& + \tilde{S}^4\left(-\frac{78913341669}{560} + \frac{2112661847355}{1792}\tilde{X} - \frac{22549638312}{7}\tilde{X}^2 + \frac{711442845528}{175}\tilde{X}^3 - \frac{38345728971}{14}\tilde{X}^4 + \frac{17731459378613}{16800}\tilde{X}^5 \right. \\
& \left. - \frac{53572666604}{225}\tilde{X}^6 + \frac{32395208551}{1050}\tilde{X}^7 - \frac{19003480}{9}\tilde{X}^8 + \frac{74038129}{1260}\tilde{X}^9 \right) + \tilde{S}^6\left(-\frac{10648440314}{35} + \frac{918947008807}{700}\tilde{X} \right. \\
& \left. - \frac{14155172678}{7}\tilde{X}^2 + \frac{9382331949013}{6300}\tilde{X}^3 - \frac{521780426341}{900}\tilde{X}^4 + \frac{254938247857}{2100}\tilde{X}^5 - \frac{1732220216}{135}\tilde{X}^6 + \frac{504318104}{945}\tilde{X}^7 \right) \\
& + \tilde{S}^8\left(-\frac{3274704127}{28} + \frac{7211558817317}{25200}\tilde{X} - \frac{5561526592 9}{225}\tilde{X}^2 + \frac{402647627639}{4200}\tilde{X}^3 - \frac{761509408}{45}\tilde{X}^4 + \frac{692470907}{630}\tilde{X}^5 \right) \\
& + \tilde{S}^{10}\left(-\frac{4342880411}{450} + \frac{12820511189}{1050}\tilde{X} - \frac{1055741186}{225}\tilde{X}^2 + \frac{3487385299}{6300}\tilde{X}^3 \right) + \tilde{S}^{12}\left(-\frac{89079832}{675} + \frac{1895391191}{37800}\tilde{X} \right),
\end{aligned}
$$

$$
\begin{aligned}
Z_6 = {} & \frac{2310525}{8}\tilde{X} - \frac{4808540583}{100}\tilde{X}^2 + \frac{252517854951}{400}\tilde{X}^3 - \frac{11473454766573}{4900}\tilde{X}^4 + \frac{143434835467311}{39200}\tilde{X}^5 - \frac{429688246765}{147}\tilde{X}^6 \\
& + \frac{9794517932561}{7350}\tilde{X}^7 - \frac{72661274793}{196}\tilde{X}^8 + \frac{2312186142587}{35280}\tilde{X}^9 - \frac{11845630792}{1575}\tilde{X}^{10} + \frac{2067628712}{3675}\tilde{X}^{11} - \frac{127687796}{4725}\tilde{X}^{12} \\
& + \frac{105557789}{132300}\tilde{X}^{13} - \frac{48128}{3675}\tilde{X}^{14} + \frac{3008}{33075}\tilde{X}^{15} + \tilde{S}^2\Bigg(- \frac{4808540583}{200} + \frac{252517854951}{200}\tilde{X} - \frac{126208002432303}{9800}\tilde{X}^2 \\
& + \frac{1864652861075043}{39200}\tilde{X}^3 - \frac{8164076688535}{98}\tilde{X}^4 + \frac{19589035865122}{245}\tilde{X}^5 - \frac{17947334873871}{392}\tilde{X}^6 + \frac{580358721789337}{35280}\tilde{X}^7 \\
& - \frac{5999811996148}{1575}\tilde{X}^8 + \frac{2104846028816}{3675}\tilde{X}^9 - \frac{86891545178}{1575}\tilde{X}^{10} + \frac{143875266407}{44100}\tilde{X}^{11} - \frac{393903616}{3675}\tilde{X}^{12} + \frac{49259008}{33075}\tilde{X}^{13}\Bigg) \\
& + \tilde{S}^4\Bigg(- \frac{11473454766573}{4900} + \frac{2438392202944287}{78400}\tilde{X} - \frac{6445323701475}{49}\tilde{X}^2 + \frac{313424573841952}{1225}\tilde{X}^3 - \frac{26376042749859}{98}\tilde{X}^4 \\
& + \frac{3928404256255313}{23520}\tilde{X}^5 - \frac{100972156871008}{1575}\tilde{X}^6 + \frac{57062417193776}{3675}\tilde{X}^7 - \frac{148756282340}{63}\tilde{X}^8 + \frac{3832908876379}{17640}\tilde{X}^9 \\
& - \frac{13467610112}{1225}\tilde{X}^{10} + \frac{2574414848}{11025}\tilde{X}^{11}\Bigg) + \tilde{S}^6\Bigg(- \frac{7304700195005}{588} + \frac{303630055909391}{3675}\tilde{X} - \frac{9736610822262}{49}\tilde{X}^2 \\
& + \frac{2078655342185713}{8820}\tilde{X}^3 - \frac{245859028495658}{1575}\tilde{X}^4 + \frac{224530004722216}{3675}\tilde{X}^5 - \frac{13559550120628}{945}\tilde{X}^6 + \frac{13054120650052}{6615}\tilde{X}^7 \\
& - \frac{107026432768}{735}\tilde{X}^8 + \frac{29281785088}{6615}\tilde{X}^9\Bigg) + \tilde{S}^8\Bigg(- \frac{2252499518583}{196} + \frac{1597720624527617}{35280}\tilde{X} - \frac{104821986878408}{1575}\tilde{X}^2 \\
& + \frac{177310534011916}{3675}\tilde{X}^3 - \frac{5960977068464}{315}\tilde{X}^4 + \frac{35848797395657}{8820}\tilde{X}^5 - \frac{328499926016}{735}\tilde{X}^6 + \frac{129412835072}{6615}\tilde{X}^7\Bigg) \\
& + \tilde{S}^{10}\Bigg(- \frac{4092665438636}{1575} + \frac{22582640792464}{3675}\tilde{X} - \frac{8264177610763}{1575}\tilde{X}^2 + \frac{180539814396049}{88200}\tilde{X}^3 - \frac{451257055744}{1225}\tilde{X}^4 \\
& + \frac{271877622784}{11025}\tilde{X}^5\Bigg) + \tilde{S}^{12}\Bigg(- \frac{697303053956}{4725} + \frac{98123248362941}{529200}\tilde{X} - \frac{263501425664}{3675}\tilde{X}^2 + \frac{284503269632}{33075}\tilde{X}^3\Bigg) \\
& + \tilde{S}^{14}\Bigg(- \frac{5592287104}{3675} + \frac{19264982656}{33075}\tilde{X}\Bigg),
\end{aligned}
$$

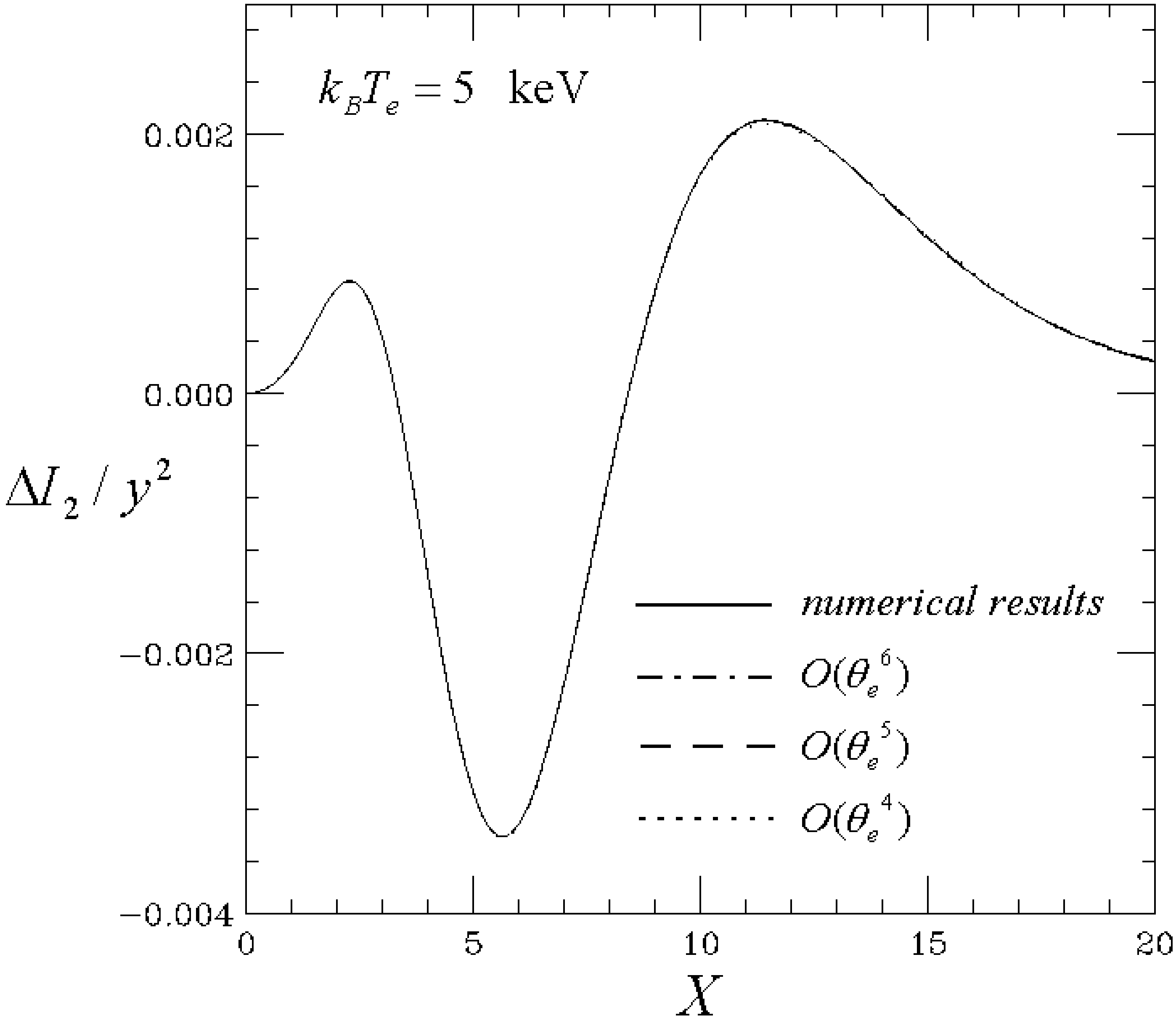
$$\Delta I = \frac{X^3}{e^X - 1}\frac{\Delta n(X)}{n_0(X)} = \Delta I_1 + \Delta I_2$$

Definition of Γ

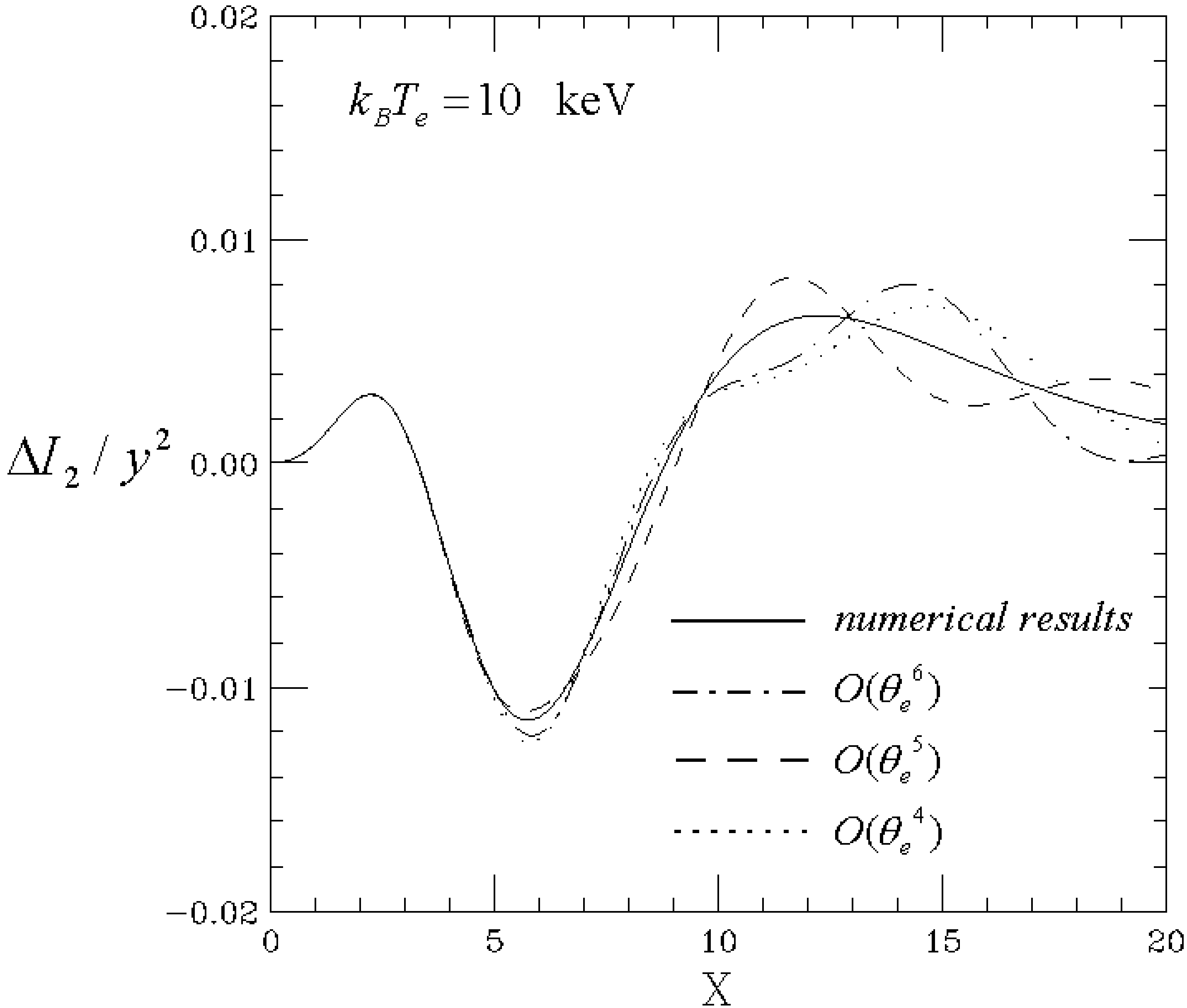
$$\Gamma \equiv \frac{\Delta I_2/y^2}{\Delta I_1/y}$$

RAYLEIGH-JEANS LIMIT

$$\frac{\Delta n(X)}{n_0(X)} = -2\,y\,\theta_e \left(1 - \frac{17}{10}\theta_e + \frac{123}{40}\theta_e^2 - \frac{1989}{280}\theta_e^3 + \frac{14403}{640}\theta_e^4 - \frac{20157}{224}\theta_e^5 + \frac{423951}{1024}\theta_e^6\right) + 2\,y^2\,\theta_e^2 \left(1 - \frac{17}{5}\theta_e + \frac{226}{25}\theta_e^2 - \frac{34527}{1400}\theta_e^3 + \frac{13758}{175}\theta_e^4 - \frac{1344789}{4480}\theta_e^5 + \frac{25927827}{19600}\theta_e^6\right).$$

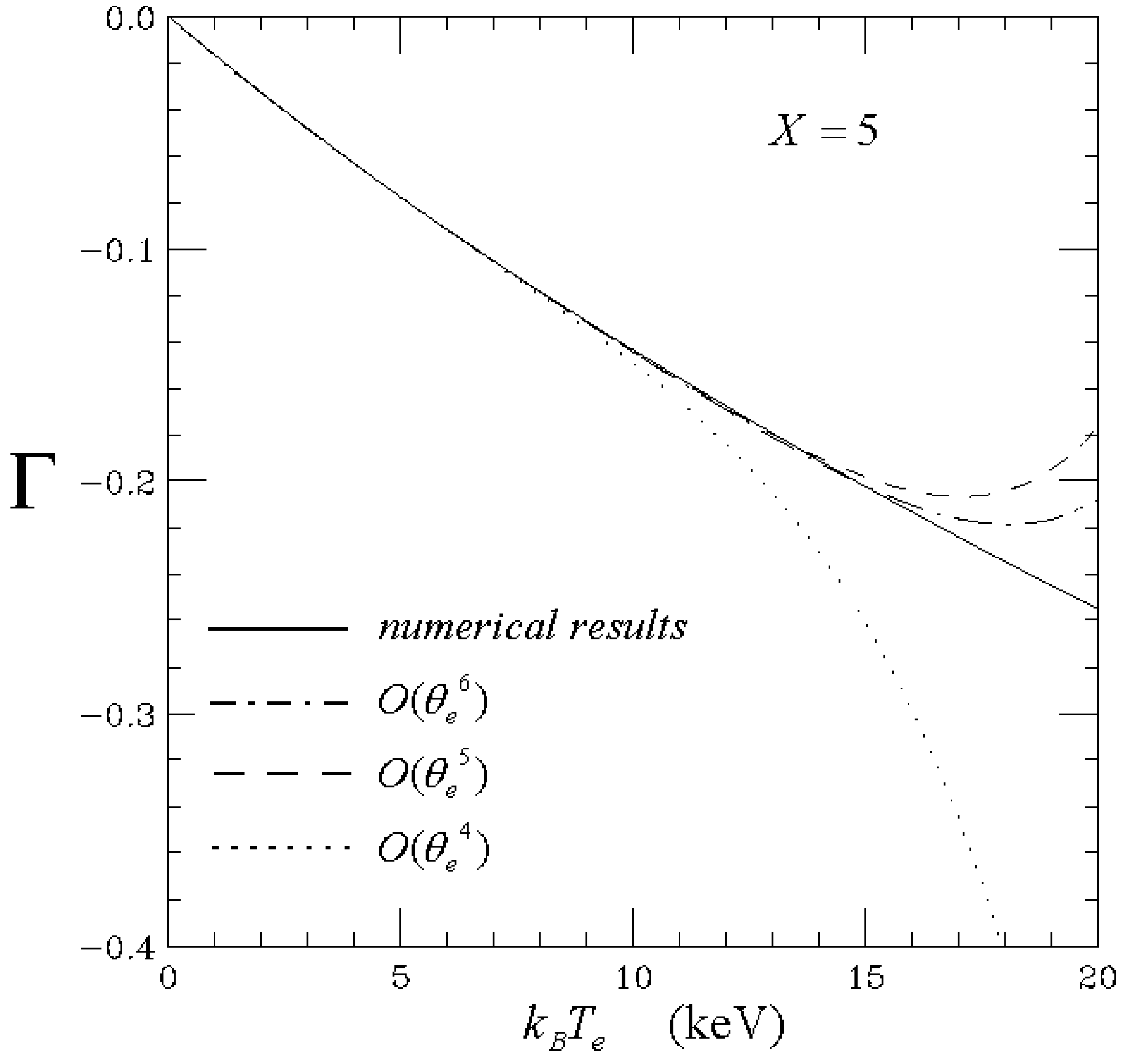


$k_B T_e = 5$ keV
0.002
0.000
−0.002
−0.004
$\Delta I_2 / y^2$
numerical results
$O(\theta_e{}^6)$
$O(\theta_e{}^5)$
$O(\theta_e{}^4)$
0
5
10
15
20
X



$k_B T_e = 10$ keV
0.02
0.01
0.00
−0.01
−0.02
$\Delta I_2 / y^2$
0
5
10
15
20
X
numerical results
$O(\theta_e^{\ 6})$
$O(\theta_e^{\ 5})$
$O(\theta_e^{\ 4})$

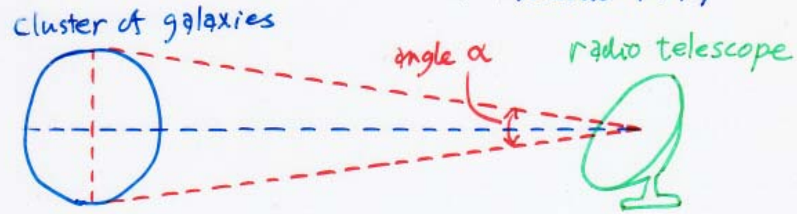
Double Scattering/Single Scattering



Gunn 1978
Silk and White 1978
Birkinshaw 1979

DETERMINATION OF THE HUBBLE CONSTANT

Cavaliere, Danese, and De Zotti 1979



Measurement of the Sunyaev-Zeldovich effect combined with the X-ray observations of Ne and Te leads to the determination of the cluster diameter.

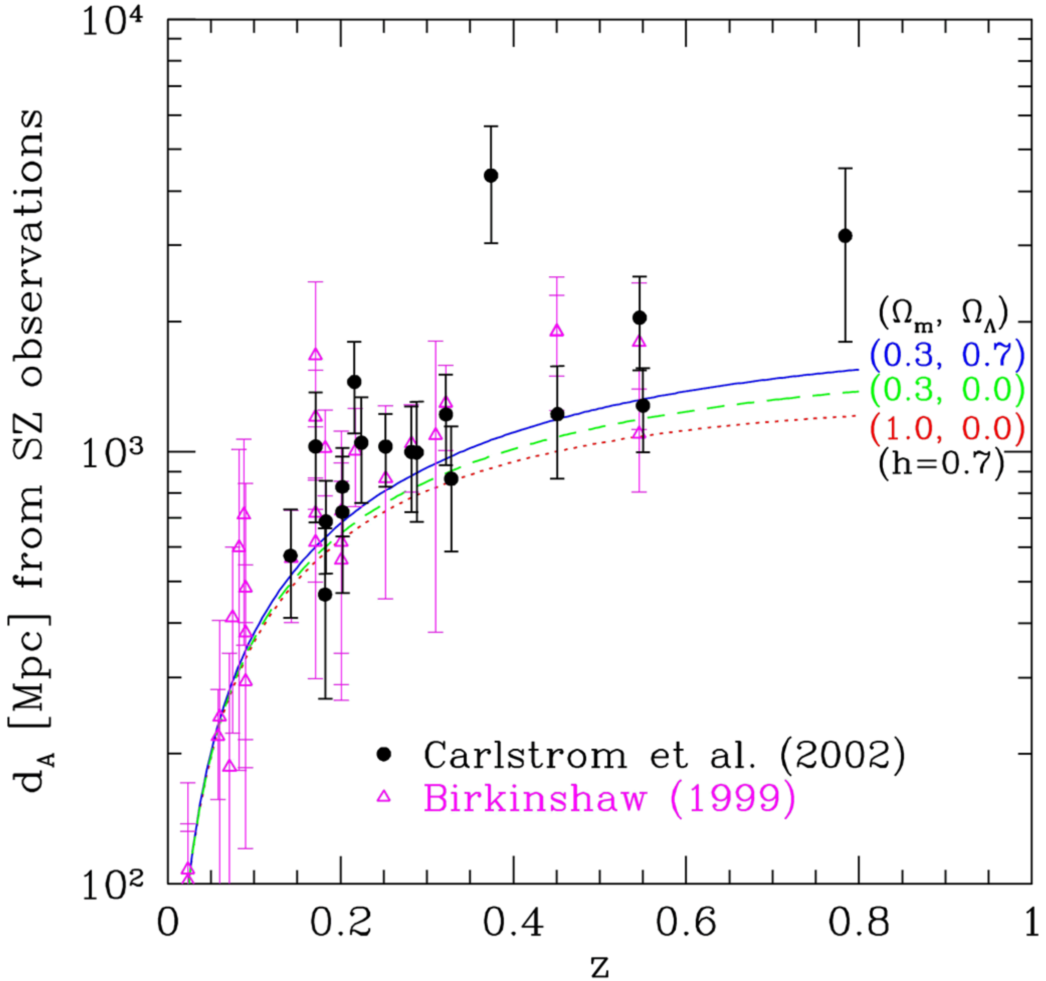
Assume spherical symmetry of the cluster.

Combined with the measurement of the angle α, this leads to the measurement of the distance to the cluster r.

Combined with the cluster red-shift (which leads to the determination of the recession velocity v), one obtains the Hubble constant H_0.

$$v = H_0 r \qquad H_0 = \frac{v}{r}$$

v: recession velocity $\quad r$: distance to the object



angular distance – red shift relation obtained from

SZ effect observations using $H_0 = 70$ km / s / Mpc

CONCLUSIONS

- Relativistic corrections to the thermal,kinematical, and polarization SZ effects have been carried out to a precision better than 1%.
- Validity of the Kompaneets equation has been rigorously assessed.
- Accurate analytic fitting formulae for the numerical results of the thermal SZ effect have been provided.
- Multiple scattering effects have been found to be much less than 1% for ordinary clusters.
- TIME IS RIPE FOR THE PRECISION SZ OBSERVATIONS, ESPECIALLY DETECTION OF THE KINEMATICAL SZ EFFECT.