

SPECTRA FROM PAIR-EQUILIBRIUM PLASMAS

ANDRZEJ A. ZDZIARSKI¹

Harvard-Smithsonian Center for Astrophysics, and Max-Planck-Institut für Physik und Astrophysik

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ABSTRACT

Spectra from relativistic Maxwellian plasmas without magnetic fields or external photon sources and in pair equilibrium are presented. Photon emission by bremsstrahlung and pair annihilation processes, multiple Compton scatterings, and photon absorption by free-free and photon-photon pair production processes are modeled numerically using the Monte Carlo technique. Optical depth effects and equilibrium effects are solved for self-consistency by iteration. Pair annihilation features are found to be absent in all spectra from pair-equilibrium plasmas. Luminosities and radii typical for active galactic nuclei, $L \approx 10^{44}$ ergs s⁻¹ at $R \approx 10^{14}$ cm, correspond to models with temperature of $kT \approx 0.5m_e c^2$ and the Thomson thickness $\tau_T \equiv (n_+ + n_-)\sigma_T R \approx 4$, where m_e is the electron mass, σ_T is the Thomson cross section, and n_+ and n_- are the positron and electron densities, respectively. The resultant flat spectra differ, however, from typical active galactic nuclear spectra with a mean X-ray spectral index of ~ 0.7 .

Subject headings: galaxies: nuclei — plasmas — radiation mechanisms — relativity — X-rays: sources

1. INTRODUCTION

Relativistic plasmas in pair equilibrium have recently been a subject of increasing interest. The condition that pair production in a plasma cloud is balanced by pair annihilation leads to a relation connecting the positron density n_+ , the temperature T , the luminosity L , and radius R (Lightman 1982; Svensson 1982, 1984; Stepney 1983a). In particular, n_+/N and T can be expressed as single-valued functions of L/R and τ_N . Here $\tau_N \equiv N\sigma_T R$, σ_T is the Thomson cross section, and N is the density of ionization electrons. Furthermore, for a plasma with $\tau_T \gtrsim 1$ the time scale for photon escape is of the order of or longer than the time scale to establish pair equilibrium. Here τ_T is the Thomson scattering depth $\tau_T \equiv (n_+ + n_-)\sigma_T R$ and n_- is the electron density. Thus, many steady cosmic X-ray and γ -ray sources are probably in pair equilibrium.

Previous work on pair equilibrium was either constrained to ultrarelativistic temperatures (Lightman 1982), or to cases with $dT/dL > 0$ (Stepney 1983a) or neglected Comptonization (Svensson 1982). On the contrary, the Monte Carlo method adopted in this work, originally developed by Pozdnyakov, Sobol', and Sunyaev (1977), can be used at any temperature and any Thomson thickness smaller than ~ 10 . This method gives also detailed spectra of the emerging radiation as well as the radiation field inside the plasma cloud.

After our numerical calculations were completed, we received a preprint by Svensson (1984), in which he treated mildly relativistic Comptonization using an analytical approach. The relations between T , L , R , and n_+/N obtained in that paper are in qualitative agreement with those presented here.

II. NUMERICAL MODEL OF A RELATIVISTIC PLASMA CLOUD

A model of a spherical plasma cloud is described by the following parameters:

- a) The dimensionless temperature $T_* \equiv kT/m_e c^2$;

- b) The dimensionless optical thickness due to ionization electrons τ_N ;
- c) The dimensionless Thomson thickness τ_T ;
- d) The radius R .

The dimensional parameter R enters only weakly, through the dependence of the low-energy spectrum on free-free absorption. The condition of pair equilibrium introduces a relation between the remaining three parameters. The Maxwellian electrons and positrons, which are assumed to have uniform spatial distributions, produce photons in the bremsstrahlung and pair annihilation processes. For the $e^\pm e^\pm$, $e^+ e^-$, and $e^\pm p$ bremsstrahlung spectrum formulae see, e.g., Svensson (1982), Stepney and Guilbert (1983), and references in those papers. For the annihilation spectrum, the fitting formula of Zdziarski (1980) has been used.

In order to model multiple photon scatterings, the Monte Carlo method similar to that of Pozdnyakov, Sobol', and Sunyaev (1977) is used. In this method, the paths of individual photons are followed. Originally, in the Pozdnyakov *et al.* paper the probability of a scattering was assumed not to depend on the scattering angle. As discussed by Lorentz (1981) and Górecki and Wilczewski (1984), this assumption leads to an additional steepening of the optically thick spectra. In this paper the correct scattering probability was used.

Concurrently to scatterings there occurs a photon absorption due to either the inverse bremsstrahlung process at the low-energy end of a spectrum (see eqs. [4] and [5]) or the photon-photon pair production process at the high energy end (see below).

As the paths of individual photons are followed during the calculations, detailed radiation field inside the cloud can be determined as well as the emerging spectra. In the actual calculations, the radiation fields at five different optical depths below the surface of a cloud are determined. This enables us to calculate the rates of the pair production in photon-photon ($\gamma\gamma$), photon-positron or electron (γe^\pm), and photon-photon ($\gamma\gamma$) collisions and opacity due to the $\gamma\gamma$ process; γe^\pm and γp opacities and the rates of the $e^\pm e^\pm$ and $e^\pm p$ pair production and $e^+ e^-$ pair annihilation can be found from the electron and positron distributions alone. For a review of the pair pro-

¹ On leave from N. Copernicus Astronomical Center, Bartycka 18, 00-716 Warsaw, Poland.

duction and pair annihilation rate formulae see, e.g., Svensson (1982, 1984). In order to avoid multidimensional integrations over photon angular distributions in the computations of the $\gamma\gamma$ pair production process, the radiation field inside the cloud was assumed to be isotropic. In such cases the optical thickness to the photon-photon pair production processes $\tau^{\gamma\gamma}$ reduces to a sum of one-dimensional integrals (see Gould and Schröder 1967; Brown, Mikaelian, and Gould 1973 for corrections).

The ratio of the total pair production rate R_+ and the pair annihilation rate R_- tells us if the plasma is in pair equilibrium. Equilibrium solutions for which $R_+ = R_-$ are found by two iteration methods. First, the parameters (a)–(d) for which the cloud is in pair equilibrium can be found iteratively, e.g., by changing τ_T in consecutive models keeping other parameters constant. Second, if the optical thickness to the $\gamma\gamma$ pair production processes exceeds unity at an energy $x \equiv h\nu/m_e c^2$, a substantial part of photons with energy x will be absorbed in $\gamma\gamma$ collisions before escape. This would cause a decrease in the radiation intensity I_x , which would, in turn, decrease the optical thickness $\tau^{\gamma\gamma}(x_1)$ at energies $x_1 > 1/x$. To account for nonlinearity of the process in the cases when $\tau^{\gamma\gamma}(x) \gtrsim 1$ at such x that I_x is still high, the following iteration method has been used:

1. In the first step the nonabsorbed radiation field $I_x^1[\alpha_x^0 \equiv 0]$ and opacity due to this field $\alpha_x^1[I_x^1]$ are calculated (see eqs. [9]–[12] of Gould and Schröder 1967).

2. The new absorbed field $I_x^2[\alpha_x^1]$ and its opacity $\alpha_x^2[I_x^2]$ are calculated.

3. In this step a mean opacity, usually $\langle \alpha_x \rangle \equiv (\alpha_x^1 + \alpha_x^2)/2$ is used to calculate $I_x^3[\langle \alpha_x \rangle]$ and $\alpha_x^3[I_x^3]$. The quantities I_x^i and α_x^i , $i = 1, 2, 3$, fulfill the relations $I_x^1 \geq I_x^2 \geq I_x^3$ and $\alpha_x^1 \geq \alpha_x^2 \geq \alpha_x^3$.

4. In most cases I_x^3 was already consistent with α_x^3 , i.e.,

$$I_x^4[\alpha_x^3[I_x^3]] \approx I_x^3. \quad (1)$$

If this condition is not fulfilled, further iteration is carried out. In the above expressions brackets denote integral dependences. We find out that pair absorption substantially changes spectra and luminosities for optically thick plasmas with $n_+ \gg N$ and $T_* \lesssim 1$ (see Figs. 2a, 2b).

We point out that the Monte Carlo method can be used in principle to calculate spectra from any plasma, whether in equilibrium or not, provided only the pair density does not change substantially during the time a photon needs to escape; that is,

$$1 \gg \frac{t^{\text{esc}}}{t^{\text{pair}}} \approx \begin{cases} \frac{3}{16} \left(\tau_T + \frac{\tau_T^2}{2} \right) |r - 1|, & T_* \ll 1, \\ \frac{3}{32} \left(\tau_T + \frac{\tau_T^2}{2} \right) |r - 1| \frac{\ln T_*}{T_*^2}, & T_* \gg 1. \end{cases} \quad (2)$$

Here $t^{\text{esc}} = R(1 + \tau_T/2)/c$, $t^{\text{pair}} = n_+/|dn_+/dt|$, and $r = R_+/R_-$. All the plasmas considered in this paper are in pair equilibrium ($r = 1$) and, of course, satisfy equation (2).

III. PLASMA IN PAIR EQUILIBRIUM: SPECTRA AND PARAMETERS

The ratio of the pair production and pair annihilation processes depends on all four parameters (a)–(d) defined in § II. Dependence on the radius, however, is only through the frequency x_0 below which emitted photons are mostly absorbed in the free-free process rather than scattered up in energy. The mean energy amplification in a single scattering can be approximated as $A \approx 1 + 4T_* + 16T_*^2$ (Svensson 1984). The mean

number of scatterings in a uniform spherical cloud with $\tau_T \gg 1$ is $\tau_T^2/5$ (Sunyaev and Titarchuk 1980) and $\sim \tau_T$ for $\tau_T \ll 1$. Thus, the condition that photons scattered up from the frequency x_0 do not affect either the luminosity or the pair production rate can be written approximately as

$$x_0 A^{(\tau_T^2/5 + \tau_T)} \ll \frac{T_*}{T_*^2 + 1}, \quad (3)$$

where for the right-hand side it follows from the results that most of the luminosity in Comptonized bremsstrahlung-annihilation spectra is at photon energies $x \approx T_*$ and the threshold energy for pair production is $x \approx 1/T_*$. Since for mildly relativistic plasmas, $A \gtrsim 2$, x_0 is determined by

$$R\alpha_{\text{ff}}(x_0) = \max(1, \tau_T), \quad (4)$$

and

$$x_0 \approx 4 \times 10^{-11} \min(\tau_T, \tau_T^{1/2}) \left[\frac{10^{14} \text{ cm} \ln(T_*/x_0)}{RT_*} \frac{1}{30} \right]^{1/2}, \quad (5)$$

(see Lightman and Band 1981). When equation (3) is satisfied, the frequency x_0 becomes unimportant, and the radius R no longer enters by itself as a determining parameter. In this case, $L \propto R$, and then it is convenient to replace L by the dimensionless quantity $l \equiv L/L_0$, where $L_0 \equiv m_e c^3 R/\sigma_T = 3.70 \times 10^{42} (R/10^{14} \text{ cm}) \text{ ergs s}^{-1}$.

Figure 1 presents dependences of τ_T and T_* on l for three different values of τ_N equal to 10^{-3} , 1, and $10^{1/2} = 3.16$. As found by Lightman (1982) and Svensson (1982), the dependence $T(L)$ is not monotonic; the temperature reaches a maximum value $T_*^{\text{max}}(L)$ and then decreases with increasing luminosity. For $\tau_N \rightarrow 0$, T_*^{max} is equal to ~ 25 . A plasma with $dT/dL < 0$ is isothermally unstable. It is stable, however, if the heating rate rather than the temperature is kept constant. In the cases when $(n_+ + n_-) \gg N$, $\tau_T \gg \tau_N$, and the thickness τ_N no longer enters as a determining parameter. For $\tau_N \lesssim 3.16$, the $T_*(l)$ and $\tau_T(l)$ functions become independent of τ_N at $l \gtrsim 30$ (see Fig. 1).

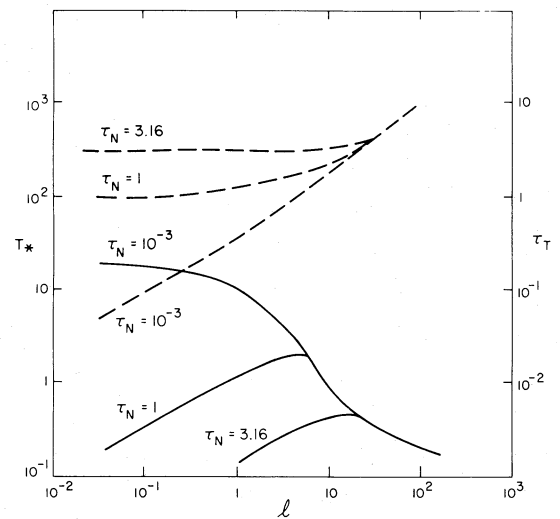


FIG. 1.—Temperature T_* (solid curves) and the Thomson thickness τ_T (dashed curves) of plasmas in pair equilibrium as functions of the luminosity for three different τ_N . The luminosity $l \equiv L/L_0$ is given in terms of $L_0 \equiv m_e c^3 R/\sigma_T = 3.70 \times 10^{42} (R/10^{14} \text{ cm}) \text{ ergs s}^{-1}$. For $l \gtrsim 30$, all the shown functions become independent of τ_N .

Figures 2a–2e present some typical spectra from plasmas in pair equilibrium. Characteristically, all the presented spectra are flat in the X-ray region, as would be expected from a bremsstrahlung emission process. As a typical active galactic nuclei (AGN) spectral index in that region is close to 0.7 (Halpern 1982; Rotschild *et al.* 1983), this indicates the importance of some other processes in AGN radiation production mechanisms. The spectra in Figures 2a–2d have been calcu-

lated for $\tau_N = 10^{-3}$ in the region where $dT/dL < 0$ (high pair density solutions). The spectrum in Figure 2e corresponds to the case with $\tau_N = 3.16$ and $dT/dL > 0$ (low pair density). In no spectra emergent from pair-equilibrium plasmas is an annihilation feature visible. At $T_* \lesssim 1$ an equilibrium plasma cloud either has negligible pair density or is optically thick, where any annihilation feature is smeared over by Compton scatterings (see Figs. 2a–2e). On the other hand, at $T_* \gtrsim 1$, where

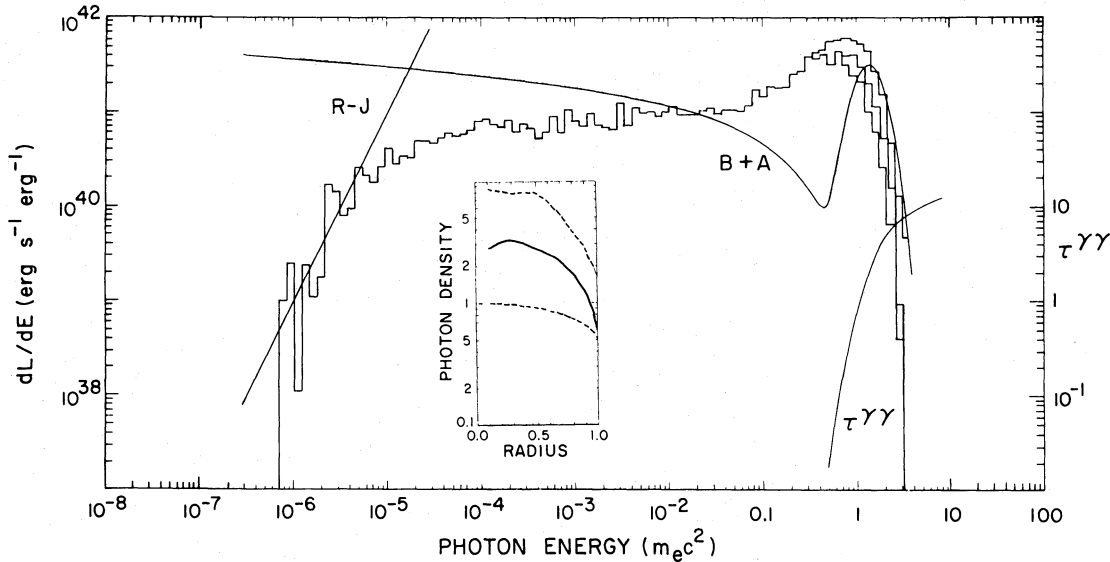


FIG. 2a

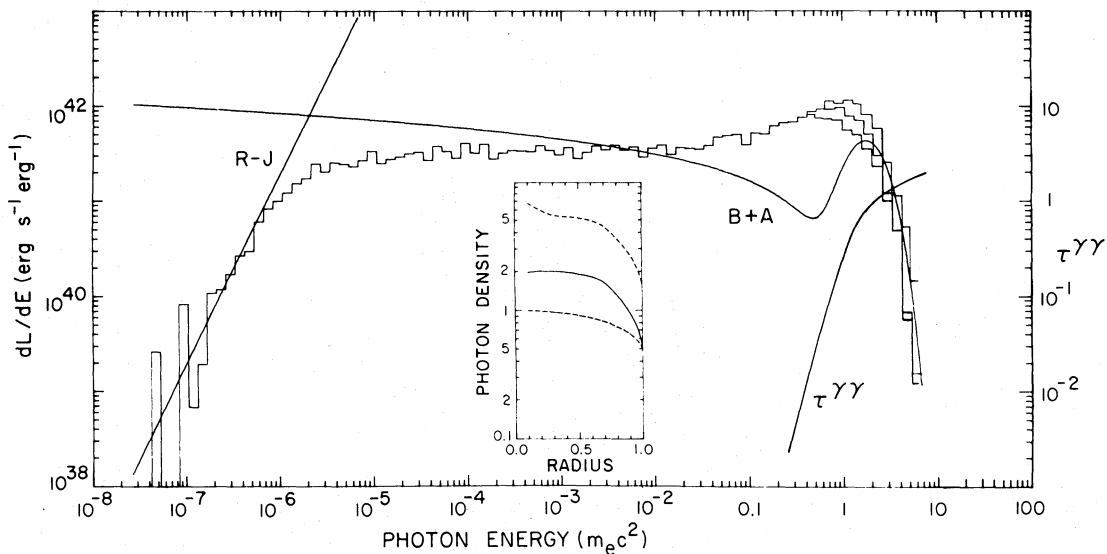


FIG. 2b

FIG. 2.—Energy spectra dL/dE (ergs s^{-1} ergs $^{-1}$) from pair-equilibrium plasmas for various temperatures T_* and scattering thicknesses τ_N and τ_T (we remind the reader, $m_e c^2 = 0.82 \times 10^{-6}$ ergs). The luminosity dL/dE is roughly proportional to the radius R (see § III). Solid curves denoted B + A are the optically thin bremsstrahlung and pair annihilation spectra, those denoted R-J are the Rayleigh-Jeans spectra. Histograms resulting from the Monte Carlo calculations include both Comptonization and absorption effects. In (a)–(b), the iterative method of computing of pair absorption is demonstrated by the upper, medium, and lower histograms, corresponding to the I_x^1 , I_x^2 , and I_x^3 radiation fields, respectively (see § II). Curves denoted $\tau_{\gamma\gamma}$ in (a)–(c) are the optical thicknesses due to pair absorption (right scale). Insets in (a)–(b) show photon densities as functions of the radius (see § III). Plasma parameters are (a) $T_* = 0.25$, $\tau_N = 10^{-3}$, $\tau_T = 8$, the radius $R = 1.5 \times 10^5$ cm, the dimensionless luminosity $l = 72$, the energy amplification factor $\alpha = 1.3$, the ratio of the pair production rates in $\gamma\gamma$ and γe^\pm collisions $R_{\gamma\gamma}/R_{\gamma e} = 2.2 \times 10^4$; (b) $T_* = 0.5$, $\tau_N = 10^{-3}$, $\tau_T = 4$, $R = 1.5 \times 10^5$ cm, $l = 27$, $\alpha = 1.8$, $R_{\gamma\gamma}/R_{\gamma e} = 820$; (c) $T_* = 1$, $\tau_N = 10^{-3}$, $\tau_T = 2.2$, $R = 1.5 \times 10^5$ cm, $l = 12$, $\alpha = 2.8$, $R_{\gamma\gamma}/R_{\gamma e} = 70$; (d) $T_* = 3.16$, $\tau_N = 10^{-3}$, $\tau_T = 1$, $R = 1.5 \times 10^6$ cm, $l = 4.5$, $\alpha = 3.0$, $R_{\gamma\gamma}/R_{\gamma e} = 7.6$; (e) $T_* = 0.3$, $\tau_N = 3.16$, $\tau_T = 3.18$, $R = 4.8 \times 10^8$ cm, $l = 4.2$, $\alpha = 9.2$.

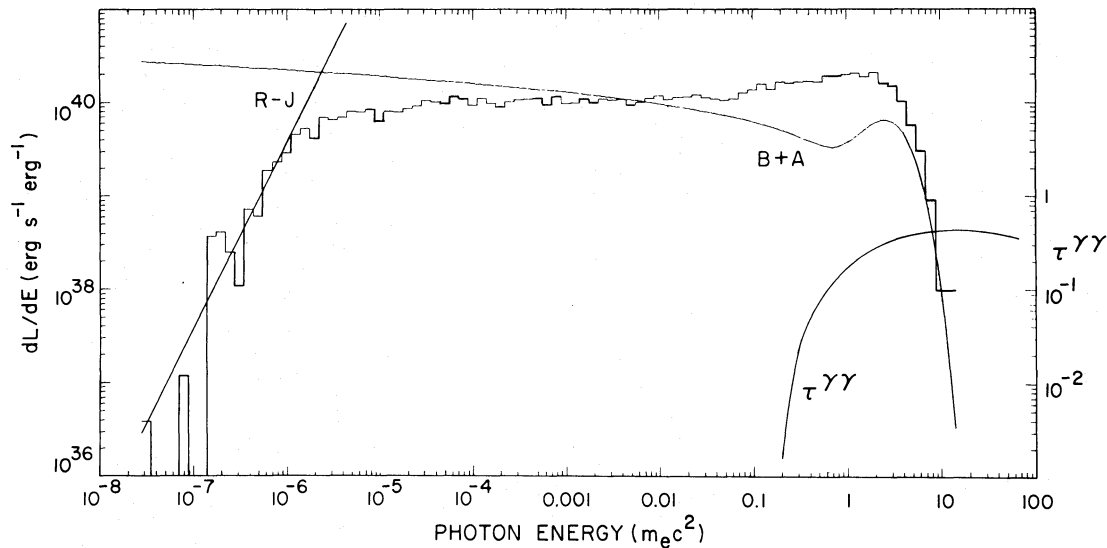


FIG. 2c

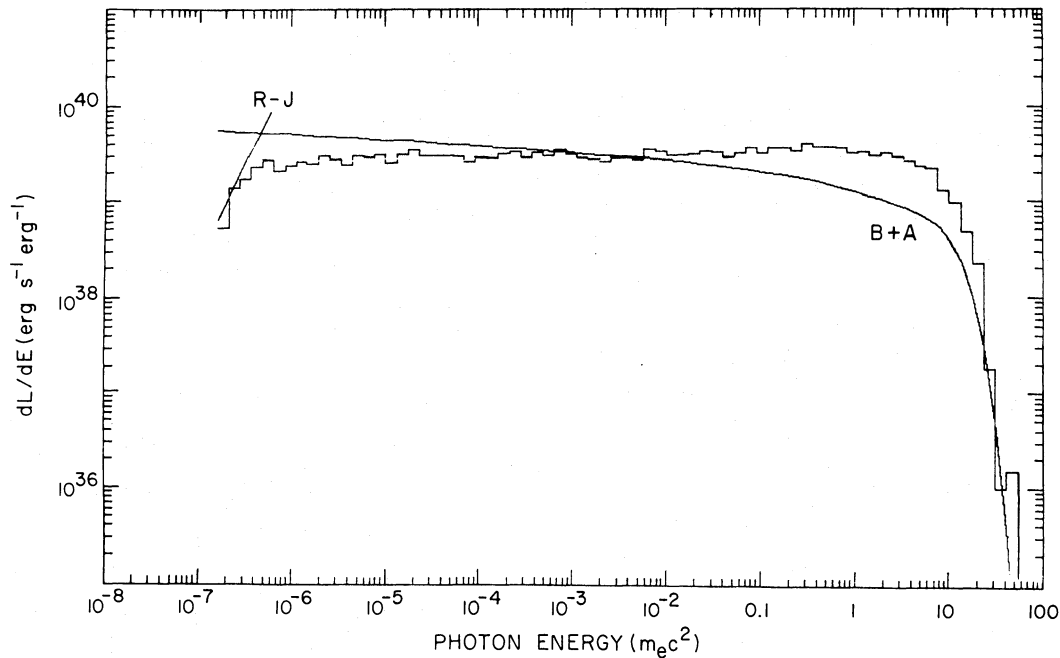


FIG. 2d

an optically thin, high pair density plasma can be in pair equilibrium, annihilation photons form a very broad feature which is indistinguishable when superposed on the bremsstrahlung spectrum (Fig. 2d). The spectra shown in Figures 2a–2b have intensity peaks at $x \sim 3T_*$, which are, however, much broader than the corresponding Wien spectra at the same temperatures. For the spectra in Figures 2a–2b, the effect of pair absorption is also shown. The upper histograms in the high-energy tails of the spectra have been calculated neglecting pair absorption (I_x^1 in the notation of § II), whereas the lower ones correspond to I_x^2 , and the medium ones are the self-consistent spectra I_x^3 . The optical thicknesses $\tau_{\gamma\gamma}$ are also plotted in Figures 2a–2c. As seen in Figures 2a–2b, pair absorption tends to decrease the contribution to the spectrum from the scattered annihilation photons. The insets in Figures 2a–2b present photon densities

as functions of the radius for (i) all photons present in the cloud (solid curves) and normalized to the density in a cloud of vanishing Thomson thickness (lower dashed curves); and (ii) for photons above the threshold for $\gamma\gamma$ pair production (upper dashed curves) normalized to the above-threshold density of a cloud of vanishing Thomson thickness. The densities (i) are substantially lower than those predicted by the relation $n_\gamma(\tau_T \gtrsim 1) = n_\gamma(0)\tau_T$. This is because the mean Thomson depth at which a photon is produced is less than τ_T for a sphere and because the Thomson cross section substantially decreases at relativistic energies. Furthermore, a correction for free-free absorption, which has not been made, would increase the total photon density (solid curves) by $\sim 30\%$. The spectra in Figures 2c–2e are characterized by zero spectral indices regardless of temperature and the optical thickness τ_T .

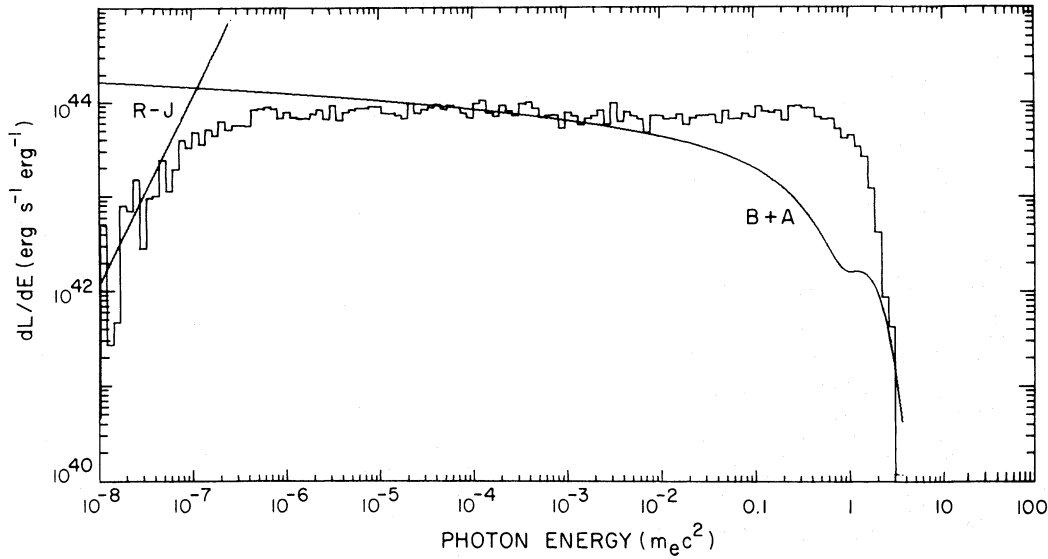


FIG. 2e

For all the spectra presented in Figure 2, $\gamma\gamma$ is the dominant pair production process; i.e. the ratio of rates of $\gamma\gamma$ to γe^\pm pair production, $R_{\gamma\gamma}/R_{\gamma e}$, is much greater than unity. For $\tau_N \ll 1$ and $dT/dL < 0$, $R_{\gamma e}$ becomes greater than $R_{\gamma\gamma}$ at $10 < T_* < 20$, while at $20 < T_* < T_*^{\max} \approx 25$ particle-particle collisions dominate (the limiting values of T_* equal to 10 and 20 obtained by us are the same as those shown in Fig. 7 by Svensson 1982).

In the cases with high τ_T , bremsstrahlung photons undergo a high amplification of energy in multiple photon scatterings. However, in the cases with $n_+ \gg N$ and $T_* \lesssim 1$, the bulk of the emitted energy is in annihilation photons, which lose energy in Compton scatterings. Consequently, the net energy amplification for pair-dominated plasmas with $T_* \lesssim 1$ is small, $l \approx (1.3 \div 2.8)(l_B + l_A)$, where l_B and l_A are the dimensionless bremsstrahlung and pair annihilation luminosities, respectively. Specifically, for $T_* \ll 1$ and $n_+ \gg N$, $l_B \ll l_A = (\pi/2)\tau_T^2$.

Figure 3 shows the time scale for cooling t_*^{cool} , and the electron-electron energy exchange time t_*^{ee} expressed in units of the Thomson time $t_T \equiv [(n_+ + n_-)\sigma_T c]^{-1}$ and as functions of the luminosity l . For the latter time scale, the results of Stepney (1983b) have been used. The cooling time t_*^{cool} is shorter than t_*^{ee} for $T_* \gtrsim 5$, as found by Stepney (1983b) and Gould (1982); Comptonization does not change this result. The quantity t_*^{cool} was found within a factor of ~ 1.5 to be a unique function of the luminosity l independently of τ_N as far as $l > 0.1$. For $l \gtrsim 100$ ($\tau_T \gtrsim 10$), the cooling time becomes shorter than the Thomson time; i.e., $t_*^{\text{cool}} < 1$.

IV. CONCLUSIONS

In this paper a numerical model of relativistic non-magnetized plasma with uniform temperature and electron density distributions was considered, and spectra from plasmas in pair equilibrium were studied. The temperature range $T_* \gtrsim 0.2$ was considered. The spectra from low pair density plasmas in pair equilibrium vary from uncomptonized bremsstrahlung spectra at $\tau_N \ll 1$ to Comptonized bremsstrahlung spectra at $\tau_N \gtrsim 1$ such as shown in Figure 2e (see Zdziarski 1984). For high pair density plasmas the spectra are flat for $T_* \gtrsim 1$ (Figs. 2c–2d) and have broad intensity peaks at $x \approx 3T_*$ for $T_* \lesssim 1$ (Figs. 2a–2b). In the latter region the total luminosity is approximately twice the annihilation luminosity. No spectra, however, reveal any annihilation feature. Pair absorption

modifies high-energy tails of optically thick high pair density spectra, tending to reduce the contribution from the scattered annihilation photons. All spectra are flat in the X-ray region, in contradiction to observed AGN spectra, with a mean spectral index of ~ 0.7 .

The cooling time, the $e-e$ relaxation time, and the Thomson time are compared in Figure 3. For the dimensionless luminosity $l \gtrsim 100$ ($\tau_T \gtrsim 10$) the cooling time becomes shorter than the Thomson time.

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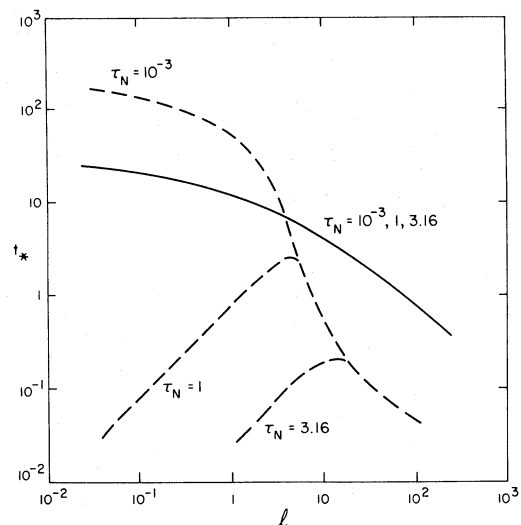


FIG. 3.—Cooling time t_*^{cool} (solid curve) and the $e-e$ relaxation time t_*^{ee} (dashed curves) in terms of the Thomson time $t_T \equiv [(n_+ + n_-)\sigma_T c]^{-1}$ as functions of the luminosity l for three different τ_N . In the shown luminosity region the t_*^{cool} is independent of τ_N within a factor of ~ 1.5 . For t_*^{ee} the results of Stepney (1983b) have been used.

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ANDRZEJ A. ZDZIARSKI: Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138